algebra form, under set-union, a cardinal algebra in which $a+a=a$.

Another interesting feature of the book is the construction, given a cardinal algebra $\mathfrak{A}$ and a group G of "partial automorphisms" of $\mathfrak{A}$, of a "refinement algebra," very similar to a cardinal algebra, in which elements of $\mathfrak{A}$ equivalent under G (directly or by decomposition) are identified. In this direction, various abstract analogs of theorems relating to the existence of measure, and to the Banach-Tarski paradox, are proved. (Let $\mathfrak{A}$ consist of Borel sets, and let G be the group of isometries of space.) The algebra of cardinal numbers under addition is deduced from the algebra of sets by the same construction.

Another section deals with the relation between "cardinal algebras" and other types of algebraic systems—especially semigroups and distributive lattices. Finally, an appendix discusses "cardinal products of isomorphism types"—that is, with the direct factorization of general algebraic systems with a binary operation and a zero. The results here are related to the monograph *Direct decompositions of finite algebraic systems*, by the author and Bjarni Jónsson.

It seems certain, to the reviewer, that the postulates and ingenious deductions of Professor Tarski will permanently enrich modern algebra—at the same time that they show once more the value of considering infinitary operations. On the other hand, it seems less clear that the particular combinations of conditions labelled by the author "cardinal algebra," "generalized cardinal algebra," and "refinement algebra" will survive without modification. At all events, the author is to be congratulated for penetrating deep into new and heretofore uncharted mathematical territory; the book is a "must" for everyone seriously interested in modern algebra or set theory.

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Theory of functions. By J. F. Ritt. Rev. ed. New York, Kings Crown Press, 1947. 10+181 pp. $3.00.

A student's introduction to the theory of functions has certain aspects in common with a youth's emergence into the adult world. Here he meets directness and subtlety, power and simplicity, beauty and rigor in quantity and proportion not previously experienced. A mathematician who undertakes to mediate this introduction by means of a book must be in control of these properties. Beyond that he needs the tact which prevents him from overwhelming the student with needless detail or deluding him with insufficient mathematical content. In the book under review these conditions have been met with ease. The teacher in search of a textbook will find, agreeably,