Cardinal algebras. By A. Tarski. New York, Oxford University Press, 1949. 12+326 pp. $10.00.

Professor Tarski's latest book is built around an axiomatic study of cardinal numbers (including zero) under finite and countable addition. Besides the commutative and associative laws in their general (countably infinite) form, these operations satisfy a Refinement Postulate ($a+b=\sum c_i$ implies the existence of a_i, b_i such that $\sum a_i=a$, $\sum b_i=b$, and $a_i+b_i=c_i$), and a Remainder Postulate (if $a_1=b_1+b_2+b_3+\cdots+b_n+a_{n+1}$ for all finite n, then c exists such that $a_1=\sum_{i<\infty}b_i+c$).

A "cardinal algebra" is a system satisfying these postulates. Among cardinal algebras, we may include: Boolean σ-algebras; the non-negative elements of any complete l-group, if $+\infty$ is adjoined; and relation numbers under cardinal addition. On the other hand, since $3|2\cdot 2\cdot 2\cdots$ (countable multiplicands) without dividing any factor, cardinal numbers do *not* form a cardinal algebra under multiplication. The first part of the book is devoted to the formal properties of such cardinal algebras. After defining $a\leqq b$ to mean that $a+x=b$ has a solution, and $n\cdot a$ as $a+\cdots+a$ (n summands), the following typical results are proved: $a\leqq b$ and $b\leqq a$ imply $a=b$ (Schroeder-Bernstein Theorem); if $a+n\cdot c\leqq b+n\cdot c$, then $a+c\leqq b+c$—whence $n\cdot a=n\cdot b$ implies $a=b$; if $a_i\leqq b_j$ for all i, j of a finite or countable set, then c exists such that $a_i\leqq c\leqq b_j$ for all i, j. Again, if $a\cap b$ exists in the sense of lattice theory, then $a\cup b$ exists and $a+b=(a\cap b)+(a\cup b)$; under other existential hypotheses, $a\cap\sum_{i<\infty}b_i=\sum_{i<\infty}(a\cap b_i)$ and $a+\bigcap_{i<\infty}b_i=\bigcap_{i<\infty}(a+b_i)$. In fact, since $\sum_{i<\infty}a_i=\text{Sup}\ \{\sum_{i<n}a_i\}$, one can define countable sums in terms of binary sums; but the postulates in terms of binary sums alone would be more awkward.

Since the proof of the postulates for addition of cardinal numbers does not involve the Axiom of Choice for uncountable sets, deductions from the postulates are also independent of this Axiom, provided no further appeal to it is made. Hence the book is an important axiomatic contribution to the foundations of set theory.

On the other hand, some readers may find it difficult to accept the author's use of "the class of all sets" (p. 215). When it comes to "generalized cardinal algebras," in which closure under the operations is not assumed, there are further logical difficulties, noted in the addenda.

However, many noteworthy theorems about generalized cardinal algebras are proved. Thus every generalized cardinal algebra can be extended to a cardinal algebra; again the ideals of any cardinal