

The geometry of the zeros of a polynomial in a complex variable. By Morris Marden. (Mathematical Surveys, no. 3.) New York, American Mathematical Society, 1949. 10+183 pp. \$5.00.

Since the time of Gauss there has been continued interest in those problems that center around the location of the zeros of a polynomial. Short expositions of the theory have been given, and a lengthier one by Dieudonné, but now, with the publication of this work, the third in the series of Mathematical Surveys of the American Mathematical Society, a comprehensive treatise on the subject has been made available. The lengthy, 20-page bibliography attests to the aim of the author to cover the material thoroughly; and considerable work must have been involved to integrate this heterogeneous material into such unity as is possible. Even so, the book could not have been kept to 161 pages had the author not adopted the plan of putting considerable theoretical material into the problems that appear at the end of most sections. Where necessary, hints are given with the problems, a wise gesture as readers of the book will see.

In a well-written preface there is a brief account of the origin of the subject, and a statement of the two central problems that the book is to treat, together with a declaration of the principal working tools. The zeros of a polynomial $f(z)$ are functions of the coefficients. Thus one problem is to specify regions, determined by these coefficients, in which the zeros lie. Again, with $f(z)$ one may associate a second polynomial (frequently this is the derivative $f'(z)$), and thus arises the problem of relating the location of the zeros of the associate to the location of those of $f(z)$. There are also variants of these problems, of which something will be said as the individual chapters are discussed. As for the methods and tools that are used, they are classical. Thus, among other theorems, those of Cauchy, Rouché, and Hurwitz on zeros of analytic functions are of considerable use; and many theorems involve, either in proof or in statement, geometric and algebraic concepts of an elementary nature.

Chapter I is introductory. The above-mentioned classical theorems are stated and proofs given. Then various interpretations, from physics, geometry, and function-theory, are given for the zeros of the rational function

$$F(z) = \sum_{j=1}^p \frac{m_j}{z - z_j}.$$

One from physics goes back to Gauss: The zeros of $F(z)$ are the equilibrium points in a force field due to p masses m_1, \dots, m_p at the