

TOPOLOGICAL METHODS IN THE THEORY OF LEBESGUE AREA

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1. **Introduction.** In conformity with the title of this address, the principal objective is the consideration of topological techniques which have been employed in the theory of surface area initiated by Lebesgue [15]¹ in his thesis.

It may appear strange that topology is required at all in a discipline classified in the field of analysis—certainly Lebesgue in 1902 gave little hint of what was to come. On the other hand, it cannot be claimed that this sort of thing is an isolated phenomenon, and perhaps one may be permitted to argue that one of the beauties of mathematics is the presence of such mixtures between fields.

Historically speaking, it was de Geöcze who called attention to certain phenomena—in a series of highly significant papers written during the years 1908 to 1914—and, in attempting to fit these phenomena into the framework of the theory, first noticed the need for topology. (For a partial list of these papers see de Geöcze [12]. A more detailed bibliography at this point, and elsewhere throughout the paper, is found in Radó [26].)

In discussing this matter let it be said, even at the risk of considerable over-simplification, that *the significant problems in Lebesgue area have their source in the attempt to find 2-dimensional analogues for four classical results in the theory of length*. In any event, with an eye on cohesion, it is here planned to consider only those topological techniques which have bearing on what might be called the *analogy problem*.

In consequence of this general plan of attack it is not only appropriate to sketch these classical results, but fortunately in so doing one automatically gains a measure of intuition by analogy and adequate motivation is provided for the 2-dimensional theory.

Before plunging into the discussion, however, a remark should be made as to the manner of presentation in contradistinction to the plan of attack. Matters of analogy leading to intuitive understanding are considered important enough to develop so as to be practically self-contained. (For example, during the first part of this paper every

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¹ Numbers in brackets refer to the bibliography at the end of the paper.