ON THE EQUICONVERGENCE OF FOURIER SERIES AND FOURIER INTEGRALS

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Given any function $f(x)$ defined for $-\infty<x<\infty$, let

$$(1) \qquad \int_0^\infty \{a(u) \cos ux + b(u) \sin ux\} du$$

be the Fourier integral (F.i.) of $f(x)$. Here

$$(2) \qquad a(u) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t) \cos utdt; \qquad b(u) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t) \sin utdt$$

are the cosine and sine transforms of $f(x)$. They exist as absolutely convergent integrals if $f\in L(-\infty, \infty)$, or in a certain generalized sense, if $f\in L^p(-\infty, \infty)$ with $1<p\leqq 2$.

In all these cases, the partial integrals of (1) are given by the formula

$$S_\omega(x, f) = \int_0^\omega \{a(u) \cos ux + b(u) \sin ux\} du$$
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} f(x+t) \frac{\sin \omega t}{t} dt,$$

but the last integral has meaning if

$$(3) \qquad \int_{-\infty}^{\infty} \frac{|f(t)|}{1+|t|} dt < \infty$$

even if the transforms (2) do not exist.

Now, it is well known that the problem of the representation of $f(x)$ as the limit of

$$(3a) \qquad S_\omega(x, f) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x+t) \frac{\sin \omega t}{t} dt$$

for $\omega\rightarrow\infty$ can be reduced to the representation of $f(x)$ by a Fourier series (F.s.). More precisely, we have the following theorem:

(A) *Given any function satisfying* (3), *and any interval* $I_a=(a, a+2\pi)$ *of length* 2π, *let* $f_a(x)$ *be the function of period* 2π *and coinciding with* $f(x)$ *in* I_a. *Let*

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