ON STRICTLY MINIMAL TOPOLOGICAL DIVISION RINGS

LEOPOLDO NACHBIN

1. **Introduction.** It is well known that, for every real or complex topological vector space, the following statements hold good:

(1) A nonvanishing linear functional on the space is continuous if and only if its kernel is closed in the space;

(2) Every finite-dimensional space possesses only one admissible topology.

In this note we shall determine the widest classes of topological division rings that can be used as scalar domains of topological vector spaces so as to preserve these important propositions. The results thus obtained generalize part of the recent work of J. Braconnier [2] and I. Kaplansky [5].[1] In a subsequent paper we shall apply these results to other related questions.

A *topological ring* is a ring endowed with an *admissible* topology, that is, a Hausdorff topology on the ring with respect to which the ring operations $x+y$, $-x$, xy are continuous. A *topological vector space* is a vector space over a topological division ring endowed with an *admissible* topology, that is, a Hausdorff topology on the vector space with respect to which the vector space operations $x+y$, λx are continuous. Throughout this note we shall understand the notion of *completeness* and *completion* for these topological systems in the sense formulated by A. Weil [8] (see also Bourbaki [1]). $\mathfrak{T}_1$ and $\mathfrak{T}_2$ being two topologies on the same point set, we write $\mathfrak{T}_1 \leqq \mathfrak{T}_2$ to denote that every set open according to $\mathfrak{T}_1$ must be open according to $\mathfrak{T}_2$; and $\mathfrak{T}_1 < \mathfrak{T}_2$ if, in addition to this, $\mathfrak{T}_1 \neq \mathfrak{T}_2$.

2. **Strictly minimal rings.** Let K be a topological division ring and $\mathfrak{T}_K$ be its admissible topology. A topology $\mathfrak{T}$ on K is said to be *admissible with respect to* $\mathfrak{T}_K$ if K endowed with $\mathfrak{T}$ is a topological vector space over K endowed with $\mathfrak{T}_K$: this means that the mapping $(x, y) \rightarrow x+y$ is continuous from $\mathfrak{T} \times \mathfrak{T}$ to $\mathfrak{T}$, and the mapping $(x, y) \rightarrow xy$ is continuous from $\mathfrak{T}_K \times \mathfrak{T}$ to $\mathfrak{T}$. Putting $y=1$ in the last condition, we see that the identity mapping $x \rightarrow x$ is continuous from $\mathfrak{T}_K$ to $\mathfrak{T}$, that is, $\mathfrak{T} \leqq \mathfrak{T}_K$. An obvious partial converse to this fact is the following: if $\mathfrak{T}$ is an admissible topology on K and $\mathfrak{T} \leqq \mathfrak{T}_K$, then $\mathfrak{T}$ is admissible with respect to $\mathfrak{T}_K$.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.