CLASSIFICATION OF 2-MANIFOLDS WITH SINGULAR POINTS

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1. **Introduction.** By a *closed 2-manifold*, or simply a 2-manifold, we mean here a two-dimensional connected finite simplicial complex every point of which has a neighborhood homeomorphic to a circular disk, that is, the interior of a circle. If there is a point not having the latter property, we call it a *singular point* of it.

In this paper, we shall give a complete classification (§2) and some properties (§3) of 2-manifolds with a single singular point. Obviously one may get one such geometrical figure by identifying certain points of a 2-manifold or several ones together. Conversely, we shall show that every such figure may be obtained in such a manner (see (2.3)).

In §4, we generalize these results to 2-manifolds with any number of singularities.

2. **The classification.** The classification lies in the investigation of the nature of the neighborhoods of the singular point. Let[1] $\cdot\mathfrak{M}^2$ have its singular point at 0. We first establish the following lemma.

LEMMA (2.1). *Any neighborhood of* 0 *is homeomorphic to a finite number, say p, of circular disks with all their centers identified. We call it a p-bundle and call* 0 *its center; and the boundaries of these p disks are simply said to be the boundary of the p-bundle.*

PROOF. Consider a simplicial subdivision $\mathfrak{K}^2$ of $\cdot\mathfrak{M}^2$. We first note that 0 must be a vertex of $\mathfrak{K}^2$. For, if 0 were an inner point of a 2-simplex, then 0 could not belong to any other simplex and hence would be an ordinary point; and if it were an inner point of a 1-simplex, then all the points of this 1-simplex would be singular points for the same reason. It is also evident that 0 cannot be a vertex of a 1-simplex unless it is a vertex of a 2-simplex.

Let 0 be a vertex of a 2-simplex $\mathfrak{R}^2$. Then there must be many 2-simplexes including $\mathfrak{R}^2$ forming a circular disk surrounding 0, as otherwise there would be two edges of singularities. Besides, 0 must be a vertex of another 2-simplex, say $'\mathfrak{R}^2$, by noting that 0 is a singular point. Hence we get another circular disk consisting of

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[1] We use the notation $\cdot\mathfrak{M}$ instead of a dot directly over $\mathfrak{M}$ for typographical convenience.