L-S-HOMOTOPY CLASSES ON THE TOPOLOGICAL IMAGE OF A PROJECTIVE PLANE

MARSTON MORSE

1. **Introduction.** Models for the L-S-(locally simple) homotopy classes of closed p-curves (p = parameterized) on any 2-manifold S have been announced in Morse [1].[1] Proofs have been given only for the case in which S is orientable. The present paper will treat the case in which S is the top. (topological) image of a projective plane. The proofs in the case of a general non-orientable surface can be given by an appropriate modification of methods of Morse [1] and of the present paper.

Recall that one writes $f \approx 0$ when f is a closed p-curve homotop. to zero. Deferring technical definitions until later sections, we can state the principal theorem as follows.

THEOREM 1.1. *Let h be a simple closed p-curve on the top. image S of a projective plane with h not ≈ 0 on S. Let $h^{(n)}$ $(n > 0)$ be a closed p-curve on S which traces h n times. Any L-S-closed p-curve f on S is in the L-S-homotopy class of $h^{(1)}$ or $h^{(3)}$ if h not ≈ 0, and of $h^{(2)}$ or $h^{(4)}$ if $h \approx 0$. No two of the p-curves $h^{(1)}$, $h^{(2)}$, $h^{(3)}$, $h^{(4)}$ are in the same L-S-homotopy class.*

For theorems on regular closed curves in the plane see Whitney, and H. Hopf. For L-S-closed curves in the plane see Morse [2] and Morse and Heins [1]. For a use of L-S-curves in studying deformation classes of meromorphic functions see Morse and Heins [2].

2. **L-S-curves and deformations.** Let C represent the unit circle on which $|z| = 1$ in the plane of the complex variable $z = u + iv$. With $z = e^{i\theta}$ on C, we assign C the *sense* of increasing θ. Let S be an arbitrary 2-manifold. A *closed p-curve* on S is a continuous mapping f of C into S such that the image of z in C is a point $f(z)$ in S. Two p-curves f_1 and f_2 are regarded as the *same* if and only if

$$f_1(z) = f_2(z)$$

for every z in C. The union of the points $f(z)$ in S as z ranges over C is called the *carrier* of f. The simplest case arises when the points $f(z)$ are in 1–1 correspondence with their antecedents z in C, and in this case f is termed *simple.*

Let f be a continuous mapping of C into S. Let λ be any sense

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[1] Numbers in brackets refer to the references cited at the end of the paper.