THE IRREGULARITY OF AN ALGEBRAIC SURFACE AND A THEOREM ON REGULAR SURFACES

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1. **Introduction.** In a recent paper [3][2] O. Zariski and the writer have shown that the arithmetic genus of a field Σ of algebraic functions of two variables can be invariantly defined with the aid of the Hilbert characteristic function associated with a pair of models of Σ. The first objective of this note is to show that the irregularity of Σ can be defined in a similar manner. The second objective is to obtain more general results on the existence of integral bases for regular surfaces than were obtained in [2].

2. **The irregularity.** We consider a field Σ of algebraic functions of two independent variables over a ground field k. The field k is assumed to be of characteristic zero and to be maximally algebraic in Σ. We use the notations and definitions of [3]. In particular, if U and V are normal models of Σ we let $\{A_m\}$ ($\{B_n\}$) denote the system of curves cut out on U (V) by the hypersurfaces of order m (n) of its ambient space. The dimension increased by one of the complete system $|A_m+B_n|$ (which system is regarded as lying on the join W of U and V) is denoted by $r(m, n)$. The transformation $T: U\rightarrow V$ is said to be proper [3, definition 2] if $T(A)$ is normal for a generic $A\in\{A_1\}$.

LEMMA 1. *If the transformation $T: U\rightarrow V$ is proper then there exists an integer n_0 such that for $n\geqq n_0$ and $m\geqq i$ the complete system $|A_m+B_n|$ cuts a complete series on the generic curve A_i of the system $|A_i|$, where i is an arbitrary positive integer.*

PROOF. Let A be a nonsingular irreducible hyperplane section of U such that $T(A)$ is normal. (Such hyperplane sections exist in view of the Bertini-Zariski theorems [7] and [9] and the fact that T is proper.) If $\bar{r}(m, n)$ is the r-function associated with the pair $(A, T(A))$ in the sense of [3] (article 2), then by formula 4.1 of [3], there exists an integer n_0 such that when $n\geqq n_0$ the function $r(m, n)$ satisfies the addition formula

$$r(m, n) = r(m-1, n) + \bar{r}(m, n). \tag{2.1}$$

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[2] Numbers in brackets refer to the bibliography.