ON ESSENTIALLY ABSOLUTELY CONTINUOUS PLANE TRANSFORMATIONS

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1. **Introduction.** Let Q be the unit square $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$, in the uv-plane (u, v are Cartesian coordinates), and let $x(u, v)$, $y(u, v)$ be real-valued continuous functions in Q. These functions determine a continuous mapping T: $x = x(u, v)$, $y = y(u, v)$, $(u, v) \in Q$, from Q into the xy-plane, where x, y are again Cartesian coordinates. If we introduce, for convenience, the complex variables $w = u + iv$, $z = x + iy$, then T appears in the form

$$T: \quad z = f(w), \qquad w \in Q, \tag{1}$$

where $f(w) = x(u, v) + iy(u, v)$. In connection with various problems in geometry and in analysis, there arises the problem of defining the concepts of *bounded variation* and of *absolute continuity* for continuous mappings T given as in (1). A detailed review of the extensive relevant literature may be found in Chapter IV.5 of the writer's book *Length and area*. This book will be referred to as LA, and will be used as general reference both for technical details and for literature. The present note is concerned with a line of thought that led to the concepts eBV (*essential bounded variation*) and eAC (*essential absolute continuity*). The theory of these concepts, as presented in LA, is based on the work of the writer and P. V. Reichelderfer. During the war years, Cesari in Italy developed an analogous theory based on apparently different basic concepts. However, as the writer has shown (see Bibliography [2]), the basic concepts used by Cesari are equivalent to those used in LA (see Bibliography). Thus the theories developed in LA and in the work of Cesari respectively can be combined into a single theory whose aim it is to provide two-dimensional concepts of bounded variation and absolute continuity comparable in scope and in utility to the corresponding classical one-dimensional concepts for functions of a single real variable. The present status of the theory seems to justify the assumption that the two-dimensional concepts eBV and eAC (see above) represent adequate generalizations of the corresponding one-dimensional concepts. However, it seems desirable to put the definitions of the two-dimensional concepts into a form which reveals the fundamental analogy with the one-

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