INVARIANTS OF INTERSECTION OF CERTAIN PAIRS OF SPACE CURVES

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Introduction. Projective invariants of certain pairs of curves and surfaces have been derived and characterized geometrically by the author and others. In this paper we shall continue these investigations by studying in ordinary space some pairs of curves intersecting at a nonsingular point with distinct tangents. For each pair of such curves we can determine an independent projective invariant by the use of neighborhoods up to and including the third order of the curves at the common point. Within these neighborhoods of the curves at the point there is no other affine invariant which is not a projective invariant.[1]

I. TWO CURVES INTERSECTING AT AN ORDINARY POINT WITH DISTINCT TANGENTS AND OSCULATING PLANES

1. **Derivation of an invariant** I. Let two curves C, $\overline{C}$ in ordinary space intersect at a nonsingular point 0 with distinct tangents t, $\bar{t}$ and osculating planes τ, $\bar{\tau}$ respectively. We shall first confine our attention to the case in which the line p of intersection of the osculating planes τ, $\bar{\tau}$ coincides with neither t nor $\bar{t}$. Let x, y, z represent the nonhomogeneous coordinates obtained from the projective homogeneous coordinates $x_1, \cdots, x_4$ by means of the relations

$$x = x_2/x_1, \qquad y = x_3/x_1, \qquad z = x_4/x_1.$$

If we choose the point 0 to be the origin, the tangents t, $\bar{t}$ and the line p to be respectively the x-, y-, and z-axes, then the power series expansions of the curves C, $\overline{C}$ in the neighborhood of the point 0 may be written in the form

$$C: \quad y = rx^3 + \cdots, \qquad z = ax^2 + \cdots; \tag{1}$$

$$\overline{C}: \quad x = \rho y^3 + \cdots, \qquad z = \alpha y^2 + \cdots. \tag{2}$$

The most general projective transformation which leaves the tangents t, $\bar{t}$ and the line p unchanged is expressed in terms of the nonhomogeneous coordinates by the equations

Received by the editors April 26, 1948.

[1] For some pairs of plane curves and surfaces in ordinary space L. A. Santaló has recently found some affine invariants which are not projective invariants. See his paper, *Affine invariants of certain pairs of curves and surfaces*, Duke Math. J. vol. 14 (1947) pp. 559–574.