illustration of this phenomenon we give the following example of an ideal with a basis which is a subset of its basic set.[10]

Let $\mathcal{J}$ be the field of all rational functions of x with transforming defined as the operation of replacing x by $x+1$. We consider the polynomials

$$y_1 - y, \qquad z^2 - y, \qquad z_1 - z. \tag{2}$$

Using Theorem IX of M.D.P. one shows easily that (2) is a basic set of a prime reflexive ideal Λ with coefficients in $\mathcal{J}$. Since the initials of the polynomials of (2) are unity, it follows that it is a basis for Λ. But the equations $z_1-z=0$, $z^2-y=0$, imply that $y_1=z_1^2=z^2=y$; so that any solution of these equations is also a solution of $y_1-y=0$. Thus z^2-y, z_1-z is itself a basis for Λ.

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[10] This example emerged during a conversation of the author with E. R. Kolchin.

INVERSIVE DIFFERENCE FIELDS

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It may happen that the functions of an analytic difference field admit not only the operation of replacing z by $z+1$, but also its inverse. Since these two operations have essentially the same properties it is to be expected that to each theorem concerning difference equations there will be a corresponding theorem, valid in fields of the type we have just described, in which the rôles of the highest and lowest transforms of the unknowns are interchanged. For example, J. F. Ritt has shown[1] that the number of ordinary manifolds of a first-order difference polynomial in the unknown y does not exceed its degree in y_1; and he observes that in fields where the inverse substitution is always possible, the number of ordinary manifolds is also limited by the degree in y.

The study of abstract difference fields enables us to apply this principle in every case; for, as we shall show, every abstract difference field can be extended to a difference field in which there exists, for every element h of the field, an element g such that h is the trans-

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[1] J. F. Ritt, *Algebraic difference equations*, Bull. Amer. Math. Soc. vol. 40 (1934) pp. 303–308.