A NOTE ON S-SPACES

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An S-space is a normal topological space in which each covering by open sets has a refinement which is star-finite, that is, each set of the refinement meets only a finite number of sets of the refinement. Thus a compact (=bicompact) space is an S-space, and an S-space is paracompact [1].[1]

In this note we discuss cartesian products in which one of the factors is an S-space. We show that if the other factor is compact, then the product is an S-space, and the dimension of the product does not exceed the sum of the dimensions of the factors. However, if both factors are S-spaces, the product need not be an S-space.

THEOREM. *Let A be an n-dimensional S-space and B an m-dimensional compact space. Then $A\times B$ is an S-space and* $\dim(A\times B)\leqq n+m$.

By the dimension of a space we mean, of course, the Lebesgue dimension (cf. [2, p. 206]).

Let $\mathfrak{W}_0$ be an arbitrary covering of $A\times B$. We are going to construct a locally-finite cell complex, D, with dim $D\leqq n+m$, a mapping f of $A\times B$ onto D, and a covering $\mathfrak{Y}$ of D such that $f^{-1}(\mathfrak{Y})$ is a refinement of $\mathfrak{W}_0$.

Let a be any point of A. Each point of $a\times B$ is contained in an open set of the form $U\times V$, U open in A, V open in B, such that $U\times V$ is contained in an open set of $\mathfrak{W}_0$. For a fixed point $a\in A$, the set of all such V's is a covering of B, and hence a finite number of them, say $V_{a,1}, V_{a,2}, \cdots, V_{a,k(a)}$, form a covering $\mathfrak{V}_a$ of B. Let U_a be the intersection of the corresponding U's.

The collection of all such sets U_a is a covering of A. Hence there is a star-finite refinement $\mathfrak{U}$ of this covering whose order is no more than $n+1$. We may also assume [2, p. 210] that $\mathfrak{U}$ is *normal*, that is, that there is a mapping ϕ of A onto $N(\mathfrak{U})$ such that each open set of $\mathfrak{U}$ is the inverse image, under ϕ, of the star of a vertex of $N(\mathfrak{U})$.

We form a covering $\mathfrak{W}$ of $A\times B$ as follows: each set U of $\mathfrak{U}$ is contained in some U_a, and with each U_a is associated a covering $\mathfrak{V}_a$ of B. Form the product of U with each set of $\mathfrak{V}_a$. The totality of these products forms $\mathfrak{W}$, and by construction, $\mathfrak{W}$ is a refinement of $\mathfrak{W}_0$.

Let θ be the mapping of $A\times B$ onto $N(\mathfrak{U})\times B$ defined by $\theta(a\times b)=\phi(a)\times b$, where ϕ is the above mapping of A onto $N(\mathfrak{U})$. Each ele-

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[1] Numbers in brackets refer to the references cited at the end of the paper.