NOTE ON A THEOREM OF DICKSON

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Though the problems concerning perfect and multiply perfect numbers are among the oldest of number theory, very little progress has been made in the investigation of these numbers. A k-ply perfect number is one for which $\sigma(n)=kn$ where $\sigma(n)$ is the sum of the divisors of n. The case $k=2$ is that of the perfect numbers. Though the form of the even perfect numbers is completely determined [1],[1] the question of whether or not there exists any odd perfect numbers is still a complete mystery. Sylvester [2] has shown that an odd perfect number must have at least five distinct prime factors, and Dickson [3] has proved that there are at most a finite number of odd perfect numbers having any given number of distinct prime factors. More generally, defining "primitive non-deficient" numbers to be those integers n for which

$$\sigma(n)/n \geqq 2 \tag{1}$$

and such that (1) does not hold for any proper divisor of n, Dickson showed that there are at most a finite number of odd primitive non-deficient numbers having a given number of distinct prime factors. In this note we shall give a simpler proof of Dickson's theorem; in fact prove a more general theorem which includes Dickson's as a special case.

DEFINITION 1. An integer n shall be called k-non-deficient (k any positive real number) if

$$\sigma(n)/n \geqq k \tag{2}$$

and k-deficient otherwise.

DEFINITION 2. An integer n shall be called primitive k-non-deficient if n is k-non-deficient, and all proper divisors of n are k-deficient.

Our generalization of Dickson's theorem may then be stated as:

THEOREM 1. *There are at most a finite number of primitive k-non-deficient numbers n such that*

(a) *if k is rational, n is relatively prime to the numerator of k; and*

(b) *the number of distinct prime factors of n is fixed.*

PROOF. We assume that there are an infinite number of such primi-

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[1] Numbers in brackets refer to the bibliography at the end of the paper.