BIBLIOGRAPHY

1. E. Miot, *Question* 3825, L'intermédiare des Mathématiciens vol. 18 (1911) p. 49.
2. A. Cunningham, *Solution de question* 3825, L'intermédiaire des Mathématiciens vol. 18 (1911) pp. 210–213.
3. L. E. Dickson, *History of the theory of numbers*, vol. 2, *Diophantine analysis*, Washington, 1920.

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SQUARE ROOTS IN LOCALLY EUCLIDEAN GROUPS

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A possible attack on the fifth problem of Hilbert is to demonstrate the existence of one-parameter subgroups in any locally Euclidean group.[1] It is known that, provided there are no "small" subgroups, some one-parameter subgroups exist. One would like to prove, however, that in a suitable neighborhood of the identity, every element is on one and only one one-parameter subgroup. If this is true, it is possible to extract square roots (that is, solve $x^2=a$ for given a) uniquely in this neighborhood, and the sequence of successive square roots a, $a^{1/2}$, $(a^{1/2})^{1/2}$, $\cdots$ converges to the identity. Conversely, it is easily seen that, if unique square roots exist, and if the sequence of square roots converge to the identity, then the one-parameter subgroups can be found. In this paper we give a new proof[2] that square roots exist in a suitable neighborhood of the identity and show, in addition, that either they are unique or small subgroups exist.

Throughout this paper we shall deal only with locally Euclidean topological groups of dimension n; consequently we may speak of an n-cell neighborhood of a point p, meaning a homeomorphic image of a Euclidean n-simplex containing the point p in its interior.

The proofs of Theorems 1 and 2 use the group property so sparingly that they can easily be restated as theorems on the involutions of a manifold.

THEOREM 1. *In a locally Euclidean group there exists a neighborhood*

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[1] The arguments outlined here are those of B. von Kerekjarto in his paper *Geometrische Theorie der zweigliedrigen kontinuierlichen Gruppen*, Abh. Math. Sem. Hamburgischen Univ. vol. 8 (1930) pp. 107–114.

[2] Cf. B. de Kerekjarto, *Sur l'existence de racines carrées dans les groupes continus*, C. R. Acad. Sci. Paris vol. 193 (1931) pp. 1384–1385.