A SIMPLE SOLUTION OF THE DIOPHANTINE EQUATION $x^3+y^3=z^2+t^2$

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1. Introduction. The problem of finding integral solutions for the equation $x^3+y^3=z^2+t^2$ was evidently proposed by E. Miot [1].[1] A. Cunningham [2] noted the obvious solutions $x=a^2$, $y=b^2$, $z=a^3$, $t=b^3$. Additional solutions, some of which were obtained by setting $x+y=l^2$, or $x+y=m^2+n^2$, and $x^2-xy+y^2=p^2+q^2$, were also given by Cunningham.

The equation has the obvious solutions $x=-y$, $z=t=0$. These are considered of little interest and are not discussed here. However, should it be desired to include these solutions, they may be given by $x=n$, $y=-n$, $z=t=0$, These same remarks apply to the general solution of $x^3+y^3=z^2$ given here as a direct consequence of the solution of $x^3+y^3=z^2+t^2$.

2. Solution. Let the equation first be written in the form

$$(x+y)[(2x-y)^2+3y^2]=4(z^2+t^2).$$

Hence, $(x+y)(2x-y+i\,3^{1/2}y)(2x-y-i\,3^{1/2}y)=4(z+it)(z-it)$.

Consider the system of equations,

$$n(x+y)=4s, \tag{1}$$

$$s(2x-y+i3^{1/2}y)=(p+mi+q3^{1/2}+ir3^{1/2})(z+it), \tag{2}$$

$$(p+mi+q3^{1/2}+ir3^{1/2})(2x-y-i3^{1/2}y)=n(z-it). \tag{3}$$

This system of equations will be satisfied in rational numbers if the following system, obtained from the system (1), (2), (3) by equating like components, is so satisfied. Hence, consider the system of equations,

$$nx+ny=4s, \tag{4}$$

$$2sx-sy-pz+mt=0, \tag{5}$$

$$sy-rz-qt=0, \tag{6}$$

$$mz+pt=0, \tag{7}$$

$$qz-rt=0, \tag{8}$$

$$2px+(3r-p)y-nz=0, \tag{9}$$

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[1] Numbers in brackets refer to bibliography given at end of paper.