Bibliography

1. J. J. Dennis and H. S. Wall, *The limit-circle case for a positive definite J-fraction*, Duke Math. J. vol. 12 (1945) pp. 255–273.
2. E. Hellinger and H. S. Wall, *Contributions to the analytic theory of continued fractions and infinite matrices*, Ann. of Math. (2) vol. 44 (1943) pp. 103–127.
3. J. F. Paydon and H. S. Wall, *The continued fraction as a sequence of linear transformations*, Duke Math. J. vol. 9 (1942) pp. 360–372.
4. W. T. Scott and H. S. Wall, *A convergence theorem for continued fractions*, Trans. Amer. Math. Soc. vol. 47 (1940) pp. 155–172.
5. ———, *On the convergence and divergence of continued fractions*, Amer. J. Math. vol. 69 (1947) pp. 551–561.
6. T. J. Stieltjes, *Recherches sur les fractions continues*, Oeuvres, vol. 2, pp. 402–566.
7. H. S. Wall and Marion Wetzel, *Quadratic forms and convergence regions for continued fractions*, Duke Math. J. vol. 11 (1944) pp. 89–102.
8. E. B. Van Vleck, *On the convergence of continued fractions with complex elements*, Trans. Amer. Math. Soc. vol. 2 (1901) pp. 205–233.

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REMARKS ON THE NOTION OF RECURRENCE

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We give in several lines a simple proof of Poincaré's recurrence theorem.

THEOREM. *Let Ω be a point set of finite Lebesgue measure, and T a one-to-one measure-preserving transformation of Ω into itself.*[1] *Let $B \subset A \subset \Omega$ be measurable sets such that, if $b \in B$, $T^n b \notin A$ for all positive integral n. Then the measure $m(B)$ of B is 0.*

PROOF. First we show that, if $i < j$, $(T^i B)(T^j B) = 0$. Suppose $c \in T^j B$; then from the hypothesis on B it follows that j is the smallest integer such that $T^{-j} c \in A$. Hence $c \notin T^i B$. Now if $m(B) = \delta > 0$, Ω would contain infinitely many disjunct sets $T^n B$, each of measure δ. This contradiction proves the theorem.

The following generalization of the above theorem is trivially obvious: The result holds if we replace the hypothesis that T is measure-preserving by the following: If $m(D) > 0$, $\limsup_i m\{T^i(D)\} > 0$.

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[1] For a discussion in probability language see M. Kac, *On the notion of recurrence in discrete stochastic processes*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 1002–1010.