AN INITIAL VALUE PROBLEM FOR A CLASS OF EQUATIONS OF MIXED TYPE[1]

STEFAN BERGMAN

1. Introduction. In connection with the consideration of second order linear partial differential equations of mixed type (that is, equations which are elliptic for some values of the independent variables and hyperbolic for others) there arises an initial value problem, namely that of expressing a solution of the equation in terms of the values of the solution and of its normal derivative on a portion of the boundary between the regions of elliptic and of hyperbolic behavior of the equation. Of particular interest are expressions in the form of single or multiple integrals representing the solution in terms of the prescribed data. These may enable us to deduce relationships between the initial data and various properties of the solution.[2]

In the present paper such a formula is derived for solutions of the following equation of mixed type:

$$\frac{\partial^2\psi}{\partial x^2} + (-x)^s \frac{\partial^2\psi}{\partial y^2} = 0 \qquad \text{for } x < 0, \tag{1a}$$

$$\frac{\partial^2\psi}{\partial x^2} - x^s \frac{\partial^2\psi}{\partial y^2} = 0 \qquad \text{for } x > 0, \tag{1b}$$

where s is any constant greater than -1. For $s=1$ our equation coincides with one which has been studied in an important investigation by Tricomi [6], see also [5], and which is met in the theory of two-dimensional flows of a compressible fluid when we investigate the streamfunction ψ in the so-called hodograph plane and make a certain simplifying assumption concerning the equation of state of the fluid. See [1, 2, 4]. We note that the equation is elliptic for $x<0$, hyperbolic for $x>0$.

We shall derive here an integral representation for the solution of (1a), (1b) in terms of the prescribed values $\psi(0, y)=T(y)$ and

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[2] As has been found in the study of equations of elliptic type and those of hyperbolic type, it is possible to exploit various integral representations of their solutions for the investigation of their properties. For various possibilities in this direction see §3 and [3, p. 140 ff.]. Numbers in brackets refer to the bibliography at the end of the paper.