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ON THE REPRESENTATION OF A FUNCTION AS A HELLINGER INTEGRAL

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We derive in this note a necessary and sufficient condition that a nondecreasing, continuous function h of a single variable x be representable as a Hellinger integral of the form $\int_0^x (df)^2/dg$. This condition was first proved by Hellinger in his dissertation [1].[1] Other proofs have been given by Hahn [2] and Hobson [3], who transform to Lebesgue integrals and make use of Lebesgue theory. Hellinger's proof and the less complicated proof given here have a certain simplicity in that they avoid reliance on these notions and even remain entirely within the range of monotone functions.

We consider nondecreasing functions of a real variable x on the interval $0 \leqq x \leqq 1$ (henceforth denoted as $[0, 1]$). For such a function $f(x)$ and a closed interval Δ with end points x_1 and x_2 ($x_1 \leqq x_2$), we define a new function $f_\Delta(x)$ to denote the length of the interval on the f-axis determined by the interval on the x-axis common to Δ and $(0, x)$. More precisely, denoting

$$f(x \pm 0) = \lim_{h \to 0} f(x \pm |h|) \qquad \text{if } 0 < x < 1,$$

$$f(0 - 0) = f(0); \qquad f(1 + 0) = f(1),$$

we define

$$f_\Delta(x) = \begin{cases} 0 & \text{if } 0 \leqq x < x_1, \\ f(x + 0) - f(x_1 - 0) & \text{if } x_1 \leqq x \leqq x_2, \\ f(x_2 + 0) & \text{if } x_2 < x \leqq 1. \end{cases} \tag{1}$$

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[1] Numbers in brackets refer to the references cited at the end of the paper.