SUMMABILITY WITH A GOVERNOR OF INTEGRAL ORDER[1]

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1. **Introduction.** In this paper the method of summability with a governor, introduced by G. Piranian [5],[2] is generalized, and the effectiveness of the new methods is compared with that of the Cesàro methods of integral order.

2. **Preliminary theorems.** We shall consider throughout this paper an arbitrary formal series

$$\text{(A)} \qquad \sum_{j=0}^{\infty} a_j.$$

Using the notation $s_n = \sum_{j=0}^{n} a_j$, we consider the standard Toeplitz transformation

$$\text{(1)} \qquad \sigma_n = \sum_{j=0}^{\infty} b_{nj} s_j.$$

The following theorem is well known.

THEOREM 1 (SILVERMAN-TOEPLITZ [8, 9]). *A necessary and sufficient condition that the transformation* (1) *be regular is that the following hold*:

$$(\alpha) \qquad \sum_{j=0}^{\infty} |b_{nj}| < M, \quad M \text{ independent of } n,$$

$$(\beta) \qquad \lim_{n\to\infty} b_{nj} = 0 \quad \text{for all } j,$$

$$(\gamma) \qquad \lim_{n\to\infty} \sum_{j=0}^{\infty} b_{nj} = 1.$$

Consider now an arbitrary sequence $\{\lambda_n\}$ of nonnegative terms, not all zero, and set $\Lambda_n = \sum_{j=0}^{n} \lambda_j$. Then $\Lambda_n > 0$ for all sufficiently large n. If, for these n, we let $b_{nj} = \lambda_{n-j}/\Lambda_n$ for $j \leqq n$ and $b_{nj} = 0$ for $j > n$, then (1) becomes

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[2] Numbers in brackets refer to the bibliography at the end of the paper.