ON THE SUMMABILITY OF CERTAIN ORTHOGONAL DEVELOPMENTS OF NONLINEAR FUNCTIONALS

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1. Introduction. Using Wiener's[1] method of integration in function space, Cameron and Martin[2] have constructed a set of closed orthonormal functionals in the space of nonlinear functionals which are Wiener measurable and whose squares are Wiener summable. It is the purpose of this paper to investigate the infinite-dimensional Abel summability of the orthogonal development of a nonlinear functional in terms of these orthogonal functionals.

The space of functions over which Wiener's integral operates is the space C of functions $x(t)$ defined and continuous on $0 \leqq t \leqq 1$ and vanishing at $t=0$. The whole space has measure unity, and measure in this space is built up in exactly the same way as ordinary Lebesgue measure, except that the definition of an interval and its measure are different.

Wiener defines an interval Q, or as he calls it, a "quasi-interval" in his space by the inequalities

$$Q: \qquad \alpha_j < x(t_j) < \beta_j, \qquad j = 1, \cdots, n,$$

where $t_1, \cdots, t_n$ is any finite set of numbers such that $0 < t_1 < t_2 < \cdots < t_n \leqq 1$. He defines the measure of Q as

$$m(Q) = [\pi^n t_1(t_2 - t_1) \cdots (t_n - t_{n-1})]^{-1/2} \int_{\alpha_n}^{\beta_n} \cdots \int_{\alpha_1}^{\beta_1} \cdot \exp\left[-\frac{\xi_1^2}{t_1} - \frac{(\xi_2 - \xi_1)^2}{t_2 - t_1} - \cdots - \frac{(\xi_n - \xi_{n-1})^2}{t_n - t_{n-1}} \right] d\xi_1 \cdots d\xi_n,$$

and shows that this definition is self-consistent and leads to a satisfactory measure for general sets, in accordance with the usual definition of Lebesgue measure. After measure is defined, the usual Lebesgue procedure gives a satisfactory definition of integral with all the usual properties except invariance under translations and simple behavior under magnifications. Measurability and summabil-

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[1] *Generalized harmonic analysis*, Acta Math. vol. 55 (1930) pp. 117–258, esp. pp. 214–224.

[2] *The orthogonal development of non-linear functionals in series of Fourier-Hermite functionals*, Ann. of Math. vol. 48 (1947) pp. 385–392.