NONLINEAR NETWORKS. III

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The object of this paper is to show that a certain system of nonlinear differential equations has one and only one periodic solution. These equations are of interest in that they describe the vibrations of a common type of electric network; therefore, the physical origin of the equations will be discussed first.

A linear network is a collection of linear inductors, linear resistors, and linear capacitors arbitrarily interconnected. If a periodic electromotive force is applied to this network, a periodic system of currents can exist, provided that the network has no free vibration of the same period. This, of course, is well known. The main theorem of this paper states that if in such a network the linear resistors are replaced by *quasi-linear* resistors, a periodic system of currents can again exist.

A quasi-linear resistor is a conductor whose differential resistance lies between positive limits. Quasi-linear resistors have extensive practical applications. No other type of nonlinearity except this type of nonlinear damping is considered here.

For example, consider a linear network with one degree of freedom. An inductor of inductance L, a resistor of resistance R, and a capacitor of capacitance S^{-1} are connected in series. The current $i(t)$ flowing in this circuit must satisfy the following differential equation:

$$L\frac{di}{dt} + Ri + S\int idt = g.$$

Here $g(t)$ is the electromotive force impressed in the circuit and is a periodic function of time.

The corresponding nonlinear equation to be studied is obtained by replacing the linear relation Ri by a function $V(i)$ which for all values of i is such that $A^{-1} \leqq V'(i) \leqq A$, where A is a positive constant.

In a general network with m degrees of freedom, if a set of m independent circuits (meshes) is chosen, any distribution of current in the network may be uniquely specified by assigning suitable values to the cyclic currents $i_1, i_2, \cdots, i_m$ flowing in these circuits. Let $g_1, g_2, \cdots, g_m$ be the electromotive forces acting in these circuits. It is convenient to introduce the electric charge variables $y_1, y_2, \cdots, y_m$ such that $i_j = y_j'$. The linear network equations may

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