THE MAYER HOMOLOGY THEORY

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1. Introduction. In 1942 W. Mayer [**4**][2] defined new homology groups based on a boundary operator whose pth power (p a prime) is zero, instead of the usual one whose square is zero. As a coefficient group an abelian group G with only elements of order p is used. The Mayer homology groups $H_{n,q}$ depend on two integers: $n \geqq 0$ and $0<q<p$. Mayer has established the topological invariance of these groups but left unsettled the question of their relation with the classical homology groups. This question is settled in this paper. The answer is embodied in the following theorem which is the main result of the paper.

THEOREM 1.1. *Let p be a prime and G an abelian group with all elements of order p. The Mayer homology groups $H_{n,q}$ (over G) are then related to the classical homology groups H_r (over G) as follows:*

(1) *If $n \equiv q-1 (\text{mod } p)$, then $H_{n,q} \approx H_r$ for $r=2(n-q+1)/p$.*

(2) *If $n \equiv -1$ (mod p), then $H_{n,q} \approx H_r$ for $r=2(n+1)/p-1$.*

(3) *In all other cases, $H_{n,q}=0$.*

Conversely, the classical groups H_r can be expressed in terms of the Mayer groups as follows:

(4) *If r is even, then $H_r \approx H_{n,q}$ provided $n-q=pr/2-1$.*

(5) *If r is odd, then $H_r \approx H_{n,q}$ provided $n=p(r+1)/2-1$.*

The theorem implies that the Mayer groups do not yield new topological invariants but lead instead to rather interesting alternative definitions of the classical homology groups.

The theorem is valid for the absolute and relative homology groups in simplicial complexes. It also remains valid for arbitrary spaces provided the Čech limiting process is used to define both $H_{n,q}$ and H_r.

The proof of the theorem is *not* obtained by a direct construction of the requisite isomorphisms but is an application of the axiomatic characterization of homology theory of Eilenberg and Steenrod (sketched in [1] and fully developed in a forthcoming book). Roughly speaking, the procedure is the following. It is shown that certain collections of the Mayer groups, suitably relabeled and together

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[1] Frank B. Jewett Fellow, 1947–1948.

[2] Numbers in brackets refer to bibliography at the end of the paper.