TOPOLOGICAL METHODS IN ABSTRACT ALGEBRA. COHOMOLOGY THEORY OF GROUPS[1]

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1. **Introduction.** The title of this article requires some explanation. The term "abstract algebra" was used to indicate that we shall deal with purely algebraic objects like groups, algebras and Lie algebras rather than topological groups, topological algebras, and so on. The method of study is also purely algebraic but is the replica of an algebraic process which has been widely used in topology, thus the words "toplogical methods" could be replaced by "algebraic methods suggested by algebraic topology." These purely algebraic theories do, however, have several applications in topology.

The algebraic process borrowed from topology is the following. Consider a sequence of abelian groups $\{C^q\}$ and homomorphisms δ,

$$C^0 \xrightarrow{\delta} C^1 \longrightarrow \cdots \longrightarrow C^{q-1} \xrightarrow{\delta} C^q \xrightarrow{\delta} C^{q+1} \longrightarrow \cdots \tag{1.1}$$

such that $\delta\delta = 0$. In each group C^q two subgroups are distinguished

$$Z^q = \text{kernel of } \delta : C^q \longrightarrow C^{q+1},$$
$$B^q = \text{image of } \delta : C^{q-1} \longrightarrow C^q$$

with the second definition completed by setting $B^0 = 0$. The condition $\delta\delta = 0$ is then equivalent with $B^q \subset Z^q$. The group

$$H^q = Z^q / B^q$$

is called the qth cohomology group of the sequence (1.1). The elements of the groups C^q, Z^q, B^q and H^q are called q-dimensional cochains, cocycles, coboundaries and cohomology classes respectively. Each cocycle $z \in Z^q$ determines a cohomology class as the coset $\{z\} = z + B^q$. Two cocycles in the same coset are called cohomologous.

The process just described can be applied in the following three instances.

I. *Groups.* Let Q be a multiplicative group and G an abelian group written additively. Assume further that Q operates on G, that is, that

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[1] The presentation has been brought up to date and includes many results obtained after 1945.