AN INITIAL VALUE PROBLEM FOR HYPERBOLIC DIFFERENTIAL EQUATIONS

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We consider the normal form of the linear partial differential equation of hyperbolic type

$$L[u] \equiv u_{xy} + au_x + bu_y + cu = d. \tag{1}$$

It is well known that if the coefficients of (1) satisfy certain continuity conditions, a unique solution $U(x,y)$ of (1) can be determined over any rectangle with sides parallel to the coordinate axes which lies entirely within the domain of continuity of the coefficients of (1) by prescribing the solution along any two adjacent sides of the rectangle.[1] No generality will be lost by assuming that the adjacent segments lie on the coordinate axes, so that a vertex of the rectangle is on the origin.

It will be shown here that for a certain sub-class of equations of this type a unique solution is obtainable by prescribing merely two partial derivatives of $u(x, y)$, one along each of two adjacent sides of the rectangle, that is, by prescribing $\partial^k u(x, y)/\partial x^k|_{x=0}$ and $\partial^m u(x, y)/\partial y^m|_{y=0}$ where k and m are any non-negative integers, instead of $u(x, 0)$ and $u(0, y)$. This result is obtained by reducing the new problem to the classic one ($k=m=0$).

It should also be noted that the result to be proved complements, in a certain sense, results of Bergman[2] on elliptic differential equations. If $a = \sum_{m,n} a_{mn}x^m y^n$, $b=\bar{a}$, $c = \sum_{m,n} c_{mn}x^m y^n$, $c_{mn} = c_{nm}$, are entire functions of two complex variables x and y where y is conjugate to x, and we write $x=X+iY$, $y=X-iY$; then (1) becomes an equation of elliptic type in the X, Y plane. It has been shown, by means of Bergman's operator method, that in order to determine the regularity domain of a solution $U = \sum_{m,n} U_{mn}x^m y^n$, $U_{mn} = \overline{U}_{nm}$, from a given subsequence $\{U_{mk}\}$, k fixed, $m=0, 1, 2, \cdots$, it is sufficient to know the subsequences $\{a_{m\nu}\}$, $\{a_{\nu m}\}$, $\{c_{m\nu}\}$, $\nu=0, 1, 2, \cdots, k$, $m=0, 1, 2, 3, \cdots$, of the coefficients in the power series expansion of a and c in (1). If, by the same change of variables, the result of the present paper is formulated as a theorem for equations of elliptic type, it becomes

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[1] See Courant and Hilbert, *Mathematische Physik*, vol. 2, pp. 315–316.

[2] See S. Bergman, *Linear operators in the theory of partial differential eqnations*, Trans. Amer. Math. Soc. vol. 53 (1945) pp. 130–155, in particular p. 141.