RESTRICTED MEASURABILITY

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1. **Introduction.** It is our aim to introduce into the general theory of measure a concept of restricted measurability[1] and to establish certain conditions under which it is equivalent to measurability in the familiar Carathéodory sense.

We follow the theory with two applications.

2. **Notations and conventions.** If C is a sequence of sets and n is an integer, we agree that

$$\bigcup_{j=0}^{-1} C_j, \qquad \bigcup_{j=0}^{n} C_j, \quad \text{and} \quad \bigcup_{j=0}^{\infty} C_j$$

are respectively 0, the union of terms of C numbered from 0 to n, and the union of all terms of C.

We agree that a family H of sets is *hereditary* if each subset of each member of the family is a member of the family and is *countably additive* if the union of members of each countable sub-family of H is a member of H. By σH we mean the union of all members of H.

If A and B are sets, we agree that $A \sim B$ is the set of points in A and not in B.

Throughout this paper we consider $\mathfrak{S}$ to be a fixed set with respect to which we make definitions.

3. **Restricted measurability.**

3.1. DEFINITION. ϕ is a *measure* if and only if ϕ is such a function on

$$\underset{\beta}{E}(\beta \subset \mathfrak{S}) \quad \text{to} \quad \underset{t}{E}(0 \leqq t \leqq \infty)$$

that

$$\phi(A) \leqq \sum_{\beta \in H} \phi(\beta)$$

whenever H is such a countable family that

$$A \subset \sigma H \subset \mathfrak{S}.$$

Since empty sums are zero, we see that, if ϕ is a measure, then:

I. $\phi(0) = 0$;

II. $\phi(A) \leqq \phi(B)$ whenever $A \subset B \subset \mathfrak{S}$;

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[1] A. P. Morse has contributed much to the development of this theory.