ON THE EXISTENCE OF STEADY GAS FLOW IN PLANE ISOTHERMAL STREAMLINE PATTERNS

R. C. PRIM

In this paper we shall investigate the possibility of steady, plane gas flows having streamlines which can be mapped conformally onto a family of parallel lines. We shall limit our consideration to flows of an ideal gas (that is, a nonviscous, thermally nonconducting gas with constant specific heats) in the absence of body forces. The investigation will include rotational as well as irrotational flows.

We shall employ the formulation of the basic gas flow equations due to Munk and Prim [1].[1] This formulation includes the most general steady flows of an ideal gas in the absence of body forces.

$$\text{div}\,[(1 - w^2)^{1/(\gamma-1)}\bar{w}] = 0, \tag{1}$$

is the continuity equation and the dynamic equation is

$$(\bar{w}\cdot\text{grad})\bar{w} + \frac{\gamma - 1}{2\gamma}(1 - w^2)\,\text{grad}\ln p = 0 \tag{1a}$$

whence the integrability condition

$$\text{curl}\left[\frac{\bar{w} \times \text{curl}\,\bar{w}}{1 - w^2}\right] = 0, \tag{2}$$

where γ denotes the adiabatic exponent, p the pressure, and $\bar{w}$ the reduced velocity vector related to the actual velocity vector $\bar{v}$ by

$$\bar{w} = \frac{\bar{v}}{v_{\text{ultimate}}} = \frac{\bar{v}}{(2\gamma/(\gamma - 1))(p/\rho) + v^2}$$

where ρ denotes the density.

Equations (1) and (2) will be referred to a plane, isothermal net ξ, η in which the squared element of arc length is given by

$$(ds)^2 = g(\xi, \eta)[(d\xi)^2 + (d\eta)^2]$$

where limitation of the net to a plane requires

$$\frac{\partial^2 \ln g}{\partial \xi^2} + \frac{\partial^2 \ln g}{\partial \eta^2} = 0. \tag{3}$$

Presented to the Society, December 29, 1947; received by the editors November 29, 1947.

[1] Numbers in brackets refer to the references cited at the end of the paper.