

ON THE RELIABILITY OF THE MEMBRANE THEORY OF SHELLS OF REVOLUTION

C. TRUESDELL¹

1. Introduction. The reliability of the membrane theory of shells is a somewhat controversial subject.² In my previous study of shells of revolution [MT] I have attempted to clarify the nature of the membrane theory as an approximate theory of elastic shells by deducing its differential equations as consequences of the three-dimensional infinitesimal theory of elasticity and of certain further assumptions, by discussing the type of boundary conditions to be used in problems concerning closed shells, and by proving the existence and uniqueness of solutions of the differential equations satisfying these boundary conditions. I developed also apparatus for quickly and efficiently finding the general solutions of the differential equations of the membrane theory for any given specific shell of revolution. Using this apparatus, in the present paper I shall show that:

1. In an open shell, or in a closed shell with a flat, sphere-like apex, the stress-resultants computed from the equations of the membrane theory will not exhibit a critical response to slight perturbations in the meridian curve, provided the curvature of the meridian curve is not changed very much.

2. In a closed shell with a pointed apex, a very slight change in the meridian curve in a very small region near the apex may entail very large changes in the stress resultants at all points of the shell, as computed from the membrane theory.

Both these results presuppose that the boundary condition at the apex is the "ring limit condition" stated at the end of §2. For a discussion of other possible boundary conditions, see §7.

These results and my previous treatment of the membrane theory show that in open shells or in closed shells with a flat, sphere-like apex, the stress resultants computed from the membrane theory

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² The inadequacy of the membrane theory in non-uniform problems for cones was noticed by Neményi, *Beiträge zur Berechnung der Schalen unter unsymmetrischer und unstetiger Belastung*, Bygningsskatske Meddelelser (Denmark) 1936. See also the example in C. Truesdell, *The membrane theory of shells of revolution*, Trans. Amer. Math. Soc. vol. 58 (1945) pp. 96–166, see pp. 117–118. This latter paper will be denoted henceforth by the letters MT.