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A ZERO-DIMENSIONAL TOPOLOGICAL GROUP WITH A ONE-DIMENSIONAL FACTOR GROUP

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As can be easily shown, if a locally compact topological group is zero-dimensional, all of its factor groups are zero-dimensional. In this note we give an example of a non locally compact zero-dimensional group with a factor group which is topologically isomorphic to the real numbers, hence one-dimensional.[1]

1. Preliminaries. Let $\{\lambda\}$ be a set of indices of cardinality c, and for each λ, let R_λ be a topological isomorph of the additive group of rational numbers. We form the weak product R of the R_λ: an element r of R is a collection $r=\{r_\lambda\}$, $r_\lambda \in R_\lambda$, such that for only a finite number of the λ's is $r_\lambda \neq 0_\lambda$. Under the definitions $r+r'=\{r_\lambda+r'_\lambda\}$, $0=\{0_\lambda\}$, R forms a group.

Now for each $r\in R$, we define $\|r\| = \sum_\lambda |r_\lambda|$. Since all but a finite number of the $r_\lambda = 0_\lambda$, this sum exists. Clearly $\|r+r'\| \leqq \|r\|+\|r'\|$, and $\|-r\| = \|r\|$, hence, as can be easily shown, $\|r\|$ defines a metric in R under the definition: the distance from r to r' is $\|r-r'\|$.

LEMMA 1. *Let $\{d_\lambda\}$ be a set of positive real numbers bounded away from zero, that is, there exists $d>0$ such thal $d_\lambda \geqq d$ for all λ. Then*

$$U = \left\{ r \;\middle|\; \sum_\lambda \left| \frac{r_\lambda}{d_\lambda} \right| < 1 \right\}$$

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[1] Cf. Bourbaki, *Topologie generale*, chap. III, p. 21, exercise 12, for an example of a totally disconnected group with a factor group topologically isomorphic to the reals. This example was pointed out to me by I. Kaplansky.