If we make this assumption it follows that $\mathfrak{M}$ annuls $\partial A/\partial y_{1r}$, where r is the order of A in y_1. Let s be the order of A in y_2. We form the resultant R of A and $\partial A/\partial y_{1r}$, considered as algebraic polynomials in y_{2s}. Since A is irreducible, and cannot be a factor of $\partial A/\partial y_{1r}$, R is a nonzero polynomial, free of y_{2s}, which is annulled by $\mathfrak{M}$. Since R is of lower efiective order than A in y_2, $\mathfrak{M}$ must be an essential singular manifold of A relative to y_2. The proof is now complete.

RUTGERS UNIVERSITY

DISTINCT REPRESENTATIVES OF SUBSETS

MARSHALL HALL, JR.

1. Introduction. Let W be a set of elements a_i $\cdot W = \{a_1, \cdots\}$ and let $U\{S_1, \cdots, S_j, \cdots\}$ be an indexed system of subsets of W. We wish to choose distinct representatives of the subsets. If $a_j = R(S_j)$ designates the representative of the subset S_j, then we require $R(S_j) \in S_j$ for all j and $R(S_j) \neq R(S_k)$ if $j \neq k$. It is to be emphasized that subsets are distinguished only by their indices and distinct subsets may contain the same elements. An obviously necessary condition for the existence of distinct representatives is:

Condition C: *Every k distinct subsets contain between them at least k distinct elements, for every finite k.* P. Hall[1] has shown that if the number of subsets is finite, condition C is also sufficient for the existence of a system of distinct representatives, or SDR as we shall abbreviate. This condition is no longer sufficient if the number of subsets is infinite. As a counter example consider $U(S_0, S_1, \cdots\}$ where $S_0 = \{a_1, a_2, \cdots\}$, $S_i = \{a_i\}$, $i = 1, 2, \cdots$. Here condition C is easily shown to hold for the subsets, but clearly no representative may be selected for S_0 which is not also a representative of some S_i.

In this paper it is shown that condition C is sufficient if every subset S_j is finite, and also an estimate on the number of systems of distinct representatives is given. This latter result is applied to Latin squares.

THEOREM 1. *Given an indexed system $U\{S_1, \cdots, S_j, \cdots\}$ of finite subsets of a set $W\{a_1, \cdots, a_i, \cdots\}$. If the subsets satisfy condi-*

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[1] P. Hall, *On representatives of subsets*, J. London Math. Soc. vol. 10 (1935) pp. 26–30.