BOOK REVIEWS

Length and area. By Tibor Radó. American Mathematical Society Colloquium Publications, vol. 30. New York, American Mathematical Society, 1948. 6+572 pp. $6.75.

Since as long ago as the days of the early Greek mathematicians, men have been interested in the areas of surfaces. Special examples were studied, and with the invention of the calculus a formula was found expressing the area of a sufficiently smooth surface by means of a well known double integral. Nevertheless, the careful study of areas of continuous surfaces has been almost entirely a product of the preceding half-century. During this time about a dozen major definitions of area have been proposed, all being in agreement when applied to sufficiently well-behaved surfaces, but often having widely different properties when considered for all continuous surfaces. Of these various definitions, the one due to Lebesgue has been most thoroughly investigated, and upon it there now rests a theory possessing a high degree of completeness. It is this theory which constitutes the principal subject matter of Radó's *Length and area.* The "length" is of course an interesting subject; but its placidity, as contrasted with area, permits a very complete discussion in much less than half the book. The pattern of this discussion furnishes a model for developing the study of the area.

The theory presented in this book is the outstanding application of analytic topology to a problem in analysis. Since both the topology and the analysis are essential, the author devotes the first chapter to "background material." First the ideas of curve and surface are briefly discussed (a full discussion occurs later) and the distinction is drawn between definitions based on measure theory and those (like Lebesgue's) based on semi-continuity. Examples show that even in elementary situations there is need for care and precision. Next there are 17 pages of résumé of important theorems of topology, and 20 pages of résumé of analysis, particularly set-functions and integrals.

Part II is largely concerned with the topology of curves and surfaces. A transformation on a Peano space to a Peano space is monotone if the inverse image of each point is connected; it is light if the inverse image of each point is totally disconnected. The "factorization theorem" is demonstrated: if $T(P)=P^*$ is continuous, there is a monotone transformation $M(P)=\mathfrak{M}$ of P onto a set $\mathfrak{M}$ and a light transformation $L(\mathfrak{M})=P^*$ such that $T=LM$. If the mappings $T_1(P_1)=P^*$, $T_2(P_2)=P^*$ are Fréchet equivalent they are also "K-