ON UNIVERSAL MAPPINGS AND FREE TOPOLOGICAL GROUPS

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It has been observed[1] that constructions so apparently different as Kronecker products, extension of the ring of operators of a module, field of quotients of an integral domain, free groups, free topological groups, completion of a uniform space, Čech compactification enter in the same frame. We intend in this paper to explain a rather general process of construction which may be applied to most of the examples quoted above.

This paper will proceed axiomatically. In fact the problem under question (problem of a "universal mapping") can be only stated after a certain number of axioms. When the method of construction has been explained we shall illustrate it by the classical example of the completion of a uniform space. For more examples the reader is referred to a forthcoming book of N. Bourbaki. The same method gives also necessary and sufficient conditions for many imbedding problems. Both topological and algebraic examples will be given. In the last part of the paper our method of construction will be applied to Markoff's theory of free topological groups.[2]

1. **Problems of universal mappings.** Given a set E it is possible to define on it certain kinds of structures, that is structure of ring, field, topological space.[3] We shall denote by S or T certain kinds of structures. A set with a structure T will be called a *T-set*: if T is the structure of group the T-sets are the groups. An isomorphism for the structure T will be called a T-isomorphism:

T-mappings. Induced structures. Given a kind of structure T it happens very often that, for every pair E_1E_2 of T-sets, there has been defined a family of mappings of E_1 into E_2 satisfying the following axioms:

A_1. *Every T-isomorphism is a T-mapping.*

A_2. *If $f:E_1\to E_2$ and $g:E_2\to E_3$ are T-mappings, then the composite*

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[1] Unpublished manuscripts of N. Bourbaki.

[2] Markoff, Bull. Acad. Sci. USSR. vol. 9 (1945) pp. 3–64.

[3] For precise definitions of the words "structure," "kind of structure," "isomorphism" see N. Bourbaki, *Theorie des ensembles* (*Résultats*), Part 10, Paris, Herrmann, 1939.