

suggested by a heuristic argument used by physicists in connection with the quantum mechanics equation $\psi'' + 8\pi^2 m h^{-2}(E - V(x))\psi = 0$ (Brillouin, Wentzel, Kramer). In chap. 9 it is proved that if $q' > 0$, $q'' \geq 0$, $q'' \leq (q')^\gamma (1 < \gamma < 4/3; x \geq x_0)$ and f is $L^2(0, \infty)$, then $f(x+0) + f(x-0) = 2 \sum c_n \psi_n(x)$, provided f is of bounded variation near x . In chap. 10 the author proves the following summability theorem. If continuous $q \rightarrow \infty$ monotonically and f is $L^2(0, \infty)$, then $f(x) = \lim_{v \rightarrow \infty} \sum_n (v + \lambda_n)^{-1} v c_n \psi_n(x)$ for every x for which $\int_0^\eta |f(x+y) - f(x)| dy = o(\eta)$, as $\eta \rightarrow 0$ (that is, almost everywhere). With the aid of the above result the expansion theorem of chap. 9 is then proved anew, which presents an analogy with the situation in the ordinary Fourier theory.

Some developments analogous to those in the book under review have been carried out in the field of partial differential equations by T. Carleman [Arkiv för Matematik, Astronomi och Fysik vol. 24 B (1934) pp. 1-7] and by the present reviewer [Ann. of Math. vol. 43 (1942) pp. 1-55; also, Rec. Math. (Mat. Sbornik) N.S. 20 (1947) pp. 365-430]. The field of partial differential equations being so much more difficult than that of ordinary differential equations, much more remains to be done. The book of Titchmarsh may serve as a useful guide in the line of investigation just mentioned.

W. J. TRJITZINSKY

The theory of functions of real variables. By L. M. Graves. New York and London, McGraw-Hill, 1946. 10 + 300 pp. \$4.00.

The theory of functions of real variables occupies a central position in present day analysis, and it is typical for graduate schools in the United States to offer students in mathematics a one year introduction to the subject. It has always been a problem to find a suitable text for such a course, needing, as it does, something shorter and crisper than one of the standard treatises. The volume under review is written with this end in view.

The first chapter is a short exposition of the ideas and methods of deductive logic. The notions of negation, conjunction and alternation of propositions are given symbolic notations, and regarded as undefined (but "generally understood") operations: a list of laws by which their use is to be governed (for example, double negation, excluded middle) is discussed. The calculus of classes is briefly described, and the student is introduced to a technique of translating English sentences involving logical and class relationships into briefer symbolic formulae. We are warned about possible paradoxes which may arise through unguarded use of these notions, and the writer de-