ON THE SUM OF CUBES

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Large capital letters A, B, $\cdots$ (without or with subscripts) will represent integers of the quadratic number field $Ra(\rho)$ where $\rho=(-1+(-3)^{1/2})/2$. Small latin letters a, b, $\cdots$ represent rational integers, and the conjugate of a number X is denoted by $\overline{X}$.

The object of this paper is to give a method for obtaining the complete rational integer solution for the diophantine equations of the form

$$\sum_{i=1}^{m} z_i^3 = 0, \qquad m > 3. \tag{1}$$

This equation with m even, $m=2n$, can be written as $\sum_{i=1}^{n}(X_i+\overline{X}_i)X_i\overline{X}_i=0$ where

$$X_i = z_{2i-1} + \rho(z_{2i-1} - z_{2i}) \tag{2}$$

and thus the problem of solving (1) in this case is reduced to that of finding all the integers x_i, X_i satisfying the equations

$$\sum_{i=1}^{n} x_i X_i \overline{X}_i = 0, \tag{3}$$

$$x_i = X_i + \overline{X}_i \qquad (i = 1, 2, \cdots, n) \tag{4}$$

and (2). When m is odd, $m=2n-1$, we solve the system (α) consisting of (3), $x_n=X_n=z_{2n-1}$ and (2), (4) for $i=1, 2, \cdots, n-1$.

The resolution of these two systems hinges on techniques developed by E. T. Bell [2],[1] being equivalent to the resolution of a system of multiplicative equations and a system of linear homogeneous equations in Ra in which the number of unknowns always exceeds the number of equations.

In solving (1) the following equations appear:

$$x_1y_1 + x_2y_2 + \cdots + x_ny_n = 0 \tag{5}$$

in which the x_i, y_i $(i=1, 2, \cdots, n)$ are $2n$ independent variables;

$$a_{i1}x_1 + \cdots + a_{in}x_n = 0 \qquad (i = 1, 2, \cdots, m \leqq n-1) \tag{6}$$

in which the n independent variables x_i are to be solved in terms of the coefficients a_{ij};

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[1] Numbers in brackets refer to references cited at the end of the paper.