SOME PROPERTIES OF CONTINUED FRACTIONS $1 + d_0 z + K(z/(1 + d_n z))$

W. J. THRON

1. **Introduction.** A continued fraction

$$b_0(z) + \frac{a_1(z)}{b_1(z)} + \frac{a_2(z)}{b_2(z)} + \cdots, \tag{1.1}$$

with nth approximant $A_n(z)/B_n(z)$, is said to correspond to the power series

$$1 + c_1 z + c_2 z^2 + \cdots \tag{1.2}$$

if the power series expansion of $A_n(z)/B_n(z)$ agrees with (1.2) up to and including the term $c_{k(n)}z^{k(n)}$, where $k(n) \to \infty$ as $n \to \infty$.

Leighton and Scott [1][1] proved that there is one and only one continued fraction of the form

$$1 + \frac{a_1 z^{k_1}}{1} + \frac{a_2 z^{k_2}}{1} + \cdots, \tag{1.3}$$

where all k_n are positive integers, which corresponds to a given power series (1.2).

The class of all continued fractions of the form

$$1 + d_0 z + \frac{z}{1 + d_1 z} + \frac{z}{1 + d_2 z} + \cdots, \tag{1.4}$$

which is studied in this paper, has the same property. This is shown in §2. In §3 convergence, and in particular uniform convergence, of continued fractions (1.4) is investigated. §4, finally, is devoted to a study of necessary conditions for the uniform convergence of (1.4) in a neighborhood of the origin.

2. **Correspondence.** A sufficient condition for the existence of a continued fraction (1.4) which corresponds to a given power series $P(z)$ of the form (1.2) is the existence of a solution for the system of formal identities

$$P_n(z) \equiv 1 + d_n z + \frac{z}{P_{n+1}(z)}, \qquad n = 0, 1, \cdots, \tag{2.1}$$

Presented to the Society, September 4, 1947; received by the editors March 19, 1947, and, in revised form, April 23, 1947.

[1] Numbers in brackets refer to the bibliography at the end of the paper.