ON THE REPRESENTATION, IN THE RING OF p-ADIC INTEGERS, OF A QUADRATIC FORM IN n VARIABLES BY ONE IN m VARIABLES[1]

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Introduction. In this paper, we relate the existence of p-adically integral, linear transformations taking a quadratic form f in m variables into a quadratic form g in n variables with the representation of g by f rationally without essential denominator. Before stating our result, we introduce some terminology and recall some known theorems on the subject.

We denote by R, R_∞, and R_p respectively the rational field, the real field, and the p-adic field for p an arbitrary, fixed prime. We also designate the ring of rational integers by J and the ring of p-adic integers by J_p. We recall the definition that a form f, with matrix in J, represents a form g, with matrix in J, *rationally without essential denominator*, if, for every positive, rational integer q, f may be taken into g by a linear transformation whose elements are rational numbers with denominators relatively prime to q.

We assume throughout this paper that any considered transformation is linear and that the matrix of any considered quadratic form is nonsingular and has elements in J, unless otherwise specified. We shall feel free to phrase theorems and proofs either in terms of the matrix of a form or in terms of the form itself.

It was proved by Helmut Hasse [1, pp. 205–224][2] that if f and g are quadratic forms with the same number of variables, the existence of transformations in all R_p and in R_∞, each taking f into g, implies the existence of such a transformation in R. He later [2, pp. 12–24] extended the theorem to the case where f and g do not necessarily contain the same number of variables.[3] Then C. L. Siegel [5, pp. 678–680] proved that if f and g contain the same number of variables, the existence of transformations in all J_p and in R_∞, each taking f into g, implies that f represents g rationally without essential denominator. We now wish to extend this theorem of Siegel to the case where f

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[2] Numbers in brackets refer to the references cited at the end of the paper.

[3] The reader is also referred to a proof by C. L. Siegel [6, p. 549].