A CHARACTERISTIC PROPERTY OF AFFINE COLLINEATIONS IN A SPACE OF K-SPREADS

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1. Introduction. In a recent paper[1] M. S. Knebelman has proved among other things that a necessary and sufficient condition which a mapping of an affinely connected space V_n upon itself shall satisfy in order that the covariant differentiation and the variation (the Lie derivative) of a tensor be interchangeable is that the mapping be an affine collineation. The present note deals with a similar problem in a space of K-spreads[2] by showing that the same condition is also characteristic of the isomorphic transformations.[3]

2. Affine collineations. Let

$$\frac{\partial^2 x^i}{\partial u^\alpha \partial u^\beta} + \Gamma^i_{jk}(x, p) p^j_\alpha p^k_\beta = 0 \qquad \left(p^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}\right) \tag{1}$$

be the partial differential equations of the K-spreads in an N-dimensional space, where $i, j, k, \cdots = 1, 2, \cdots, N$; $\alpha, \beta, \cdots = 1, 2, \cdots, K$. The integrability conditions are assumed to be satisfied, namely,

$$R^i_{\cdot jkl} p^j_\alpha p^k_\beta p^l_\gamma = 0,$$

where we have placed

$$\begin{aligned} R^i_{\cdot jkl} = & \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \frac{\partial \Gamma^i_{jl}}{\partial x^k} - (\Gamma^i_{jk}\big|^\tau_m \Gamma^m_{nl} - \Gamma^i_{jl}\big|^\tau_m \Gamma^m_{nk}) p^n_\tau \\ & + \Gamma^i_{nl}\Gamma^n_{jk} - \Gamma^i_{nk}\Gamma^n_{jl}, \end{aligned} \tag{2}$$

and

$$A^{\cdots}_{\cdots}\big|^\sigma_l = \partial A^{\cdots}_{\cdots} / \partial p^l_\sigma.$$

The conditions satisfied by the functions $\xi^i(x)$ such that the infinitesimal transformation

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[1] M. S. Knebelman, *On the equations of motions in a Riemann space*, Bull. Amer. Math. Soc. vol. 51 (1945) pp. 682–685.

[2] J. Douglas, *Systems of K-dimensional manifolds in an N-dimensional space*, Math. Ann. vol. 105 (1931) pp. 707–733.

[3] E. T. Davies, *On the isomorphic transformations of a space of K-spreads*, J. London Math. Soc. vol. 18 (1943) pp. 100–107.