

$$b_1 = b_2 = \cdots = b_n = 0, \beta = 0, \rho_2 = 0, \epsilon = 1 \quad \text{of V and VI,}$$

and

$$a_1 = a_2 = \cdots = a_n = 0, \alpha = 0, \rho = 0, e = 1, \lambda = 1 \quad \text{of VII,}$$

are due to G. Szegő.

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SOME REMARKS ON POLYNOMIALS

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This note contains some disconnected remarks on polynomials.

Let $f_n(x) = \prod_{i=1}^n (x - x_i)$, $-1 \leq x_1 \leq x_2 \leq \cdots \leq x_n \leq 1$. Denote by $-1 \leq y_1 \leq \cdots \leq y_{n-1} \leq 1$ the roots of $f'_n(x)$. We prove the following theorem.

THEOREM 1. *For all n*

$$(1) \quad |f_n(-1)| + |f_n(+1)| + \sum_{i=1}^{n-1} |f_n(y_i)| \leq 2^n.$$

For $n \geq 3$

$$(2) \quad |f_n(-1)|^{1/2} + |f_n(+1)|^{1/2} + \sum_{i=1}^{n-1} |f_n(y_i)|^{1/2} \leq 2^{n/2}.$$

For $n \geq n_0(k)$

$$(3) \quad |f_n(-1)|^{1/k} + |f_n(+1)|^{1/k} + \sum_{i=1}^{n-1} |f_n(y_i)|^{1/k} \leq 2^{n/k}.$$

REMARK. If $y_i = y_{i+1}$ or $-1 = y_1$, $+1 = y_{n-1}$ the corresponding summands clearly vanish.

Clearly

$$\begin{aligned} |f_n(-1)| &\leq (1 - x_1)2^{n-1}, & |f_n(y_i)| &\leq |y_i - x_{i+1}| 2^{n-1}, \\ |f_n(+1)| &\leq (1 - x_n)2^{n-1}. \end{aligned}$$

Thus