tially the two following properties of $\gamma(Z, W)$:

(a) $\gamma(Z, W)$ has at $Z=W$ a logarithmic pole;

(b) $\gamma(Z, W)$ is a symmetric harmonic function;

while the third and most restrictive property of $\gamma(Z, W)$, that is, that it vanishes on the boundary C of B, was only used in the transformation of (21) into (21′). It might be more convenient for practical computations to use instead of $\gamma(Z, W)$ simply the singularity function $-\log|z-w|$ which satisfies both conditions (a) and (b). The representation of the Neumann function is in this case a little more complicated, but on the other hand one is saved the effort of computing the Green's function of the domain with respect to Laplace's equation.

Finally, we wish to point out that we dealt with a particular type of the partial differential equation (1) only for the sake of a simple representation of the method. We indicated in our previous paper the possible generalizations of the type of equation, and for all those partial differential equations obtained one may construct the Green and Neumann functions in an analogous way.

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THE ISOPERIMETRIC PROBLEM OF BOLZA WITH FINITE SIDE CONDITIONS

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1. Introduction. In view of recent improvements in the theory of the problem of Bolza in the calculus of variations, both with respect to the necessary [**2**, **6**][1] and the sufficient conditions [**3**, **4**, **5**, **7**], it seems worthwhile to investigate certain problems which, although not immediate special cases of the problem of Bolza, can be transformed by simple means into such problems. A comparatively simple problem of this nature is that of finding necessary conditions and sufficient conditions in order that an arc

$$C: \qquad a_h,\ y_i(x) \qquad (h = 1, \cdots, r;\ i = 1, \cdots, n;\ x_1 \leqq x \leqq x_2)$$

will minimize an expression of the form

$$(1.1) \qquad I(C) = g(a) + \int_{x_1}^{x_2} f(a, x, y, y')dx$$

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[1] Numbers in brackets refer to the bibliography at the end of the paper.