BOOK REVIEWS

The theory of Lie groups, I. By Claude Chevalley. (Princeton Mathematical Series, no. 8.) Princeton University Press, 1946. 9+217 pp. $3.00.

In this masterpiece of concise exposition, the concept of Lie group is put together with all the craftsmanship of an expert. The finished product is a fascinating thing to contemplate, equipped as it is with three inter-related structures—algebraic, topological and analytic. It has no loose ends, no doubtful regions: it can be explored freely without the usual necessity of having to stay within a safe distance of the identity.

The reviewer was particularly struck that so much has been accomplished in a volume of such modest size. One of the things which makes this possible is the effectiveness of the definitions—invariably they emphasize the property which can be most quickly put to work and which is most suitable to the logic of the situation. This insistence on calling things by their right names can be disconcerting at times since it tends to ignore intuitive meanings. A tangent vector at the point p on an analytic manifold, for example, is defined as a certain type of mapping of the family of functions which are analytic at p. But in the end, this book should be easier for most readers, and far more satisfying, than any exposition of the same material proceeding on a less exact level; it should be a rugged base for volumes II, III, $\cdots$ which are to follow.

The book begins with a brief account of the orthogonal, unitary and symplectic groups of matrices and the generation of these groups by their infinitesimal elements. Then comes the important chapter on topological groups. This includes a remarkably sophisticated treatment of the covering concept. A covering of the space $\mathfrak{V}$ is a pair $(\tilde{\mathfrak{V}}, f)$ where f is a mapping $\tilde{\mathfrak{V}} \rightarrow \mathfrak{V}$ satisfying certain conditions of evenness. The coverings of topological groups are themselves converted into topological groups in a natural manner. A connected, locally connected space is called simply connected if it has no coverings other than itself (with f the identity mapping). If $\mathfrak{V}$ admits a covering $(\tilde{\mathfrak{V}}, f)$ in which $\tilde{\mathfrak{V}}$ is simply connected, it admits only one and therefore the group of automorphisms of $\tilde{\mathfrak{V}}$ which preserve f is determined only by $\mathfrak{V}$. This, by definition, is the Poincaré group of $\mathfrak{V}$. An existence theorem shows that a connected, locally connected space in which every point has a simply connected neighborhood has a simply connected covering. The author formulates and proves for simply connected spaces a