NOTE ON AFFINELY CONNECTED MANIFOLDS

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The aim of this note is to prove some statements concerning the differential geometry in the large of an affinely connected manifold.

Let M be an orientable differentiable manifold of dimension n and class two. We say that an affine connection is defined in M, if a set of quantities[1] Γ^k_{ij} is defined in each allowable coordinate system x^i such that under change of the allowable coordinate system they are transformed according to the following law:

$$\bar{\Gamma}^r_{pq} = \frac{\partial^2 x^k}{\partial \bar{x}^p \partial \bar{x}^q} \frac{\partial \bar{x}^r}{\partial x^k} + \Gamma^k_{ij} \frac{\partial x^i}{\partial \bar{x}^p} \frac{\partial x^j}{\partial \bar{x}^q} \frac{\partial \bar{x}^r}{\partial x^k}. \tag{1}$$

The connection may be symmetric or asymmetric.

It is well known that from Γ^k_{ij} the covariant derivative of a contravariant vector X^i can be defined as follows:

$$X^i_{,k} = \frac{\partial X^i}{\partial x^k} + X^l \Gamma^i_{lk}. \tag{2}$$

We also recall that the affine curvature tensor is given by

$$R^i_{jkl} = \frac{\partial \Gamma^i_{jl}}{\partial x^k} - \frac{\partial \Gamma^i_{jk}}{\partial x^l} - \Gamma^i_{ml}\Gamma^m_{jk} + \Gamma^i_{mk}\Gamma^m_{jl}. \tag{3}$$

We put

$$R_{kl} = R^i_{ikl} \tag{4}$$

and introduce the exterior differential form

$$P = R_{kl}dx^k dx^l. \tag{5}$$

Then the main theorem of this note can be stated as follows:

THEOREM. *The integral of P over any two-dimensional cycle is equal to zero.*

To prove this theorem we consider n linearly independent contravariant vectors $X^i_{(1)}, \cdots, X^i_{(n)}$ and their determinant

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[1] All indices in this paper run from 1 to n and we agree as usual that repeated indices mean summation.