SOME GENERALIZED HYPERGEOMETRIC POLYNOMIALS

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1. **Introduction.** We shall obtain some basic formal properties of the hypergeometric polynomials

$$(1)\qquad \begin{aligned} f_n(a_i;\, b_j;\, x) &\equiv f_n(a_1, a_2, \cdots, a_p;\, b_1, b_2, \cdots, b_q;\, x) \\ &\equiv {}_{p+2}F_{q+2}\left[\begin{matrix} -n, & n+1, & a_1, \cdots, a_p; \\ 1/2, & 1, & b_1, \cdots, b_q; \end{matrix}\; x\right] \end{aligned}$$

(n a non-negative integer) in an attempt to unify and to extend the study of certain sets of polynomials which have attracted considerable attention. Some special cases of the $f_n(a_i;\, b_j;\, x)$ are:[1]

(a) $f_n(1/2;\, —;\, x) = P_n(1-2x)$ (Legendre).
(b) $f_n(1;\, —;\, x) = [n!/(1/2)_n]P_n^{(-1/2,1/2)}(1-2x)$ (Jacobi).
(c) $f_n(1,\, 1/2;\, b;\, x) = [n!/(b)_n]P_n^{(b-1,1-b)}(1-2x)$ (Jacobi).
(d) $f_n(1/2,\, \zeta;\, p;\, v) = H_n(\zeta,\, p,\, v)$ [12].
(e) $f_n[1/2,\, (1+z)/2;\, 1;\, 1] = F_n(z)$ [3].
(f) $f_n(1/2;\, 1;\, t) = Z_n(t)$ [4].
(g) $f_n[1/2,\, (z+m+1)/2;\, m+1;\, 1] = F_n^m(z)$ [8].

2. **A generating function.** Let $G(y)$ be analytic at $y=0$,

$$G(y) = \sum_{n=0}^{\infty} c_n y^n,$$

and define $f_n(x)$ by the relation

$$(2)\qquad \frac{1}{1-w}\, G\left[\frac{-4xw}{(1-w)^2}\right] = \sum_{n=0}^{\infty} f_n(x) w^n.$$

If w is sufficiently small, the left member of (2) may be expanded in an absolutely convergent double series and rearranged so as to give a convergent power series in w. Let that be done. Then it is easily shown that

$$(3)\qquad f_n(x) = \sum_{r=0}^{n} \frac{(-n)_r (n+1)_r c_r x^r}{(1/2)_r r!}$$

in which $(a)_r = a(a+1)\cdots(a+r-1)$; $(a)_0 = 1$.

Received by the editors September 16, 1946, and, in revised form, February 24, 1947.

[1] A dash indicates the absence of parameters. A number in brackets relates to the references.