4.1 **Remarks.** Suppose that the hypothesis of a theorem involves convergence (σ). It may happen (and does indeed in the case of all theorems in [S]) that convergence (σ_1) may be substituted in the hypothesis as follows: each σ-region which occurs in the proof is a σ_n-region for some n; if there is a largest such n call it N. Then convergence (σ_N) may be substituted in the hypothesis and may, by Theorem I, be replaced by convergence (σ_1). Lemma 10 is an example of an exception to this statement; there is no largest n and convergence (σ) is essential.

If a series is convergent (σ_1) then, for a given n, the series is convergent (σ_n). Hence, given $\epsilon>0$ there are indices (p, q) such that $|A-\sum(R)|<\epsilon$ for every σ_n-region R which is a region (p, q). In general (p, q) will depend on n; if the choice does not depend on n then the series is convergent (σ).

BROWN UNIVERSITY

THE DIFFERENTIABILITY AND UNIQUENESS OF CONTINUOUS SOLUTIONS OF ADDITION FORMULAS

NELSON DUNFORD AND EINAR HILLE

The problem of representing a one-parameter group of operators (that is, a family T_ξ, $-\infty<\xi<\infty$, of bounded linear operators on a Banach space which satisfies $T_{\xi+\zeta}=T_\xi T_\zeta$) reduces according to several well known methods of attack to establishing differentiability of the function T_ξ at $\xi=0$. The derivative $Ax=\lim_{\xi\to 0}\xi^{-1}(T_\xi-I)x$ exists as a closed operator with domain $D(A)$ dense, providing T_ξ is continuous in the *strong* operator topology (that is, $\lim_{\xi\to\xi_0}T_\xi x=T_{\xi_0}x$, $x\in\mathfrak{X}$). It is then possible to assign a meaning to exp (ξA) in a natural way and so that $T_\xi=\exp(\xi A)$, $-\infty<\xi<\infty$. The operator A is bounded if and only if T_ξ is continuous in ξ in the *uniform* operator topology (that is, $\lim_{\xi\to\xi_0}|T_\xi-T_{\xi_0}|=0$) in which case $A=\lim_{\xi\to 0}\xi^{-1}(T_\xi-I)$ exists in the uniform topology. This implies that T_ξ is an entire function of ξ; conversely, if T_ξ is analytic anywhere, then A is bounded. These considerations extend to the semi-group case in which $T_{\xi+\zeta}=T_\xi T_\zeta$ is known to hold only for positive values of the parameters, although

Presented to the Society, August 14, 1944; received by the editors February 17, 1947.