ON THE REPRESENTATION OF σ-COMPLETE BOOLEAN ALGEBRAS

L. H. LOOMIS

A σ-complete Boolean algebra is a Boolean algebra in which for every sequence of elements a_i, $i=1, \cdots$, there is an element $\bigcup_1^\infty a_n$, the countable union of the a_i, such that $a_i \subseteq \bigcup_1^\infty a_n$ for every i, and such that if $a_i \subseteq x$ for every i then $\bigcup_1^\infty a_n \subseteq x$. The dual operation, countable intersection, can be introduced through complementation, and the distributive law $a \cap \bigcup_1^\infty a_n = \bigcup_1^\infty (a \cap a_n)$ and its dual can be proved (see [**3**, p. 93]).[1] Certain types of Boolean algebras have representations as algebras of point sets, the representation preserving all the operations of the algebra. Among these are ordinary Boolean algebras with no further operations (Stone [**1**, p. 106]) and complete Boolean algebras for which very general operations and distributive laws are assumed (Tarski [**2**, pp. 197–198]). On the other hand it is well known that a σ-complete Boolean algebra has in general no such representation. For example, the quotient of the algebra of Lebesgue measurable subsets of $[0, 1]$ modulo the ideal of sets of measure zero is a σ-complete Boolean algebra which is not σ-isomorphic to any σ-complete Boolean algebra of point sets. The following theorem, which we shall prove in this note, shows that this example illustrates the general situation.

THEOREM. *Every σ-complete Boolean algebra is σ-isomorphic to a σ-complete Boolean algebra of point sets modulo a σ-ideal in that algebra.*

In particular, every abstract measure algebra can be considered as an algebra of point sets modulo sets of measure zero. Bischof [**4**] has recently obtained a proof of this special case of the theorem, but his proof leans heavily on the existence of a measure and does not generalize.

If we are given a representation of an abstract collection R of objects onto a family F of sets, then each point p of the representation space determines a subfamily of sets F_p and hence a subcollection R_p of R, namely, those objects in R whose image sets in F contain p. This suggests the way to define points in attempting to build up a representation. A point will be a certain kind of subset of R, the image $\mathfrak{a}$ of a will be the set of points containing a, and the representation $\mathfrak{R}$ of R

Received by the editors February 12, 1947.

[1] Numbers in brackets refer to the bibliography at the end of the paper.