ON RELATIONS EXISTING BETWEEN TWO KERNELS OF THE FORM $(a, b)+b$ AND $(b, a)+b$

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Let s and t be variables in the interval from 0 to 1, and let $a, b, c, \cdots$, be functions of s and t. Putting, as is customary,

$$(a, b) = \int_0^1 a(s\lambda)b(\lambda t)d\lambda,$$

we have

$$(a \pm b, c) = (a, c) \pm (b, c),$$
$$(a, b \pm c) = (a, b) \pm (a, c),$$
$$((a, b), c) = (a, (b, c)) = (a, b, c).$$

From this follows readily the meaning of (a, b, c, d). Putting, again,

$$[a, b] = a + (a, b) + b,$$

we have

$$[0, a] = a, \qquad [a, 0] = a, \qquad [[a, b], c] = [a, [b, c]] = [a, b, c].$$

We put finally,

$$\{a, b, c\} = (a, b, c) + (a, b) + (b, c) + b.$$

The function a is said to be reciprocable if there exists a function $\bar{a}$ such that

$$(*) \qquad [a, \bar{a}] = 0 \quad \text{and} \quad [\bar{a}, a] = 0.$$

(Each of these equations, it is well known, implies the other.) We say then that $\bar{a}$ is the reciprocal of a. If a is reciprocable, then so is $\bar{a}$, and the reciprocal of $\bar{a}$ is a. In what follows we shall designate the Fredholm determinant of a function a by D_a, and the reciprocal of a by $\bar{a}$. Of the various relationships that exist among the symbols (a, b), (a, b, c), $[a, b]$, $[a, b, c]$ and $\{a, b, c\}$, we state here the following:

$$(1) \qquad [a, b, c] = \{a, b, c\} + [a, c],$$

$$(2) \qquad [a, b, \bar{a}] = \{a, b, \bar{a}\}.$$

The following relations also hold true:
(α) $\{a, b, 0\} = (a, b)+b$ $\{0, a, b\} = (a, b)+a$ $\{a, 0, b\} = 0$,

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