ON THE NON-SUMMABILITY $(C, 1)$ OF FOURIER SERIES

M. L. MISRA

Let

$$\phi(t) \sim \sum_{n=1}^{\infty} a_n \cos nt, \qquad \phi(0) = 0, \tag{1}$$

be the Fourier series associated with an even function $\phi(t)$ which is integrable (L) over the interval $(0, \pi)$ and defined outside this range by periodicity.

Lebesgue [2, pp. 561–562][1] proved that the series (1) is summable $(C, 1)$ to zero at the point $t=0$ if, as $t\to 0$,

$$\int_0^t |\phi(t)| dt = o(t). \tag{2}$$

If, however, a condition weaker than (2) is satisfied, namely

$$\int_0^t \phi(t)dt = o(t), \tag{3}$$

as $t\to 0$, the series (1) is not necessarily summable $(C, 1)$. Hahn [1] gave an example to prove that the series (1) is not summable $(C, 1)$, though the condition (3) is satisfied but not (2). Prasad [3] has investigated whether, at the point $t=0$, the series (1) would be summable $(C, 1)$ if a condition stronger than (3) is satisfied, namely

$$\int_0^\delta \frac{\phi(t)}{t} dt = \lim_{t\to 0} \int_t^\delta \frac{\phi(t)}{t} dt = s, \tag{4}$$

say, exists as a non-absolutely convergent integral, and he has constructed an example to show that even when (4) is satisfied the series (1) is not necessarily summable $(C, 1)$. As the condition (4) implies (3), Prasad's example includes that of Hahn.

The object of the present note is to construct an example to prove that the series (1) is not necessarily summable $(C, 1)$ even though the condition

$$\phi_1(t) \equiv \int_t^\pi \frac{\phi(t)}{t} dt - s = o\left(1/\log\log\frac{1}{t}\right), \tag{5}$$

as $t\to 0$, is satisfied.

Received by the editors October 22, 1946.

[1] Numbers in brackets refer to the references cited at the end of the paper.