BOOK REVIEWS

Lectures on the calculus of variations. By G. A. Bliss. University of Chicago Press, 1946. 9+296 pp. $5.00.

Calculus of variations is in the main the study of properties of a real-valued function on a class $\mathfrak{M}$ with particular attention to maximizing and minimizing properties of certain elements in $\mathfrak{M}$. Normally the elements of $\mathfrak{M}$ are curves or surfaces embedded in an euclidean space. There are two main problems in the calculus of variations, problems in the large and problems in the small. The first of these is concerned with existence theorems of minimizing elements of various types, the classification and the counting of these elements and the connections between these minimizing elements and the topology of $\mathfrak{M}$. The calculus of variations in the small is concerned primarily with the properties of a particular minimizing element and the determination of those properties that will insure a minimum. Besides these two aspects, there is the fascinating study of the relations between the calculus of variations and differential equations, geometry, physical applications, and the like.

The book under review is concerned with the problem in the small. It contains the best introduction to the calculus of variations known to the reviewer, no matter what phase of the subject one wishes to pursue. The book undoubtedly will become the standard text for the beginner and will be used by the specialist as a source of material and ideas.

Professor Bliss restricts himself to the case in which the class $\mathfrak{M}$ referred to above is a class of continuous arcs C: $y_i(x)$ $(x_1 \leqq x \leqq x_2)$ $(i=1, \cdots, n)$ in euclidean $(x, y_1, \cdots, y_n)$-space. These arcs are assumed to have a piecewise continuously turning tangent with elements (x, y, y') in a prescribed region R of (x, y, y')-space. Professor Bliss begins first with the case when the end values $[x_1, y(x_1), x_2, y(x_2)]$ are fixed and develops the theory in a simple and direct manner. Later he permits these end values to lie on a prescribed end manifold. Finally he requires the arcs to satisfy in addition a given set of differential equations $\phi_\beta(x, y, y')=0$ $(\beta=1, \cdots, m<n)$. The function to be minimized is of the form

$$I(C) = g[x_1, y(x_1), x_2, y(x_2)] + \int_{x_1}^{x_2} f(x, y, y')dx,$$

with $g\equiv 0$ in the fixed end point case. By this procedure the author begins with the simplest problem in the calculus of variations and