SEQUENCES OF IDEAL SOLUTIONS IN THE TARRY-ESCOTT PROBLEM

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1. Introduction. In the Tarry-Escott problem (sometimes called the problem of multi-degree equalities, or of equal sums of like powers), one seeks integral solutions of the k equations

$$\sum_{i=1}^{s} x_i^l = \sum_{i=1}^{s} y_i^l, \qquad l = 1, 2, \cdots, k. \tag{1}$$

The usual notation is to represent a solution of (1) by

$$a_1, \cdots, a_s \overset{k}{=} b_1, \cdots, b_s. \tag{2}$$

Such a solution is called *trivial* if the a's form a permutation of the b's, and will be called *semi-trivial* if any $a_i = a_j$ or $b_i = b_j$. A solution is said to be in *reduced* form when $\sum a_i = 0$, $(a_i, b_i) = 1$, and solutions having the same reduced form are called *equivalent* solutions.

It is easily shown that for nontrivial solutions, $s > k$. The case $s = k+1$, called the *ideal* or *optimum* case [1, 2],[1] is of particular interest in many applications [5]. For a given k, $N(k)$ is defined as the least value of s for which (1) has nontrivial solutions. It is known in general [7] that $N(k) \leqq k(k+1)/2$, but numerical examples [6] give $N(k) = k+1$ for $k = 1, 2, \cdots, 9$.

In order to decrease the number of the equations (1), many writers have imposed the conditions

$$x_i = -y_i, \qquad i = 1, 2, \cdots, s, \text{ for } s \text{ odd}, \tag{3}$$

or

$$x_{s+1-i} = -x_i, \quad y_{s+1-i} = -y_i, \qquad i = 1, 2, \cdots, s/2, \text{ for } s \text{ even}. \tag{4}$$

Solutions of (1) subject to (3) or (4) will be called *symmetric* solutions. It is evident that the conditions for symmetry are sufficient to assure that symmetric solutions are reduced.

By use of the binomial theorem, one finds that (2) implies

$$Ma_1 + K, \cdots, Ma_s + K \overset{k}{=} Mb_1 + K, \cdots, Mb_s + K, \tag{5}$$

and

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[1] Numbers in brackets refer to the references cited at the end of the paper.