free. An elementary counting process used in the proof yields an asymptotic formula for a number $S(n)$ of square-free numbers of the form ax^2+b, $x \leqq n$. The method of proof can be used for several generalizations. (Received January 7, 1947.)

119. R. D. Wagner: *The generalized Laplace equations in a function theory for commutative algebras.*

The author considers the generalization of the theory associated with the Laplace equation $\partial^2 u/\partial x^2+\partial^2 u/\partial y^2=0$ in the theory of functions of a complex variable to that of functions in a commutative linear associative algebra with a principal unit. The usual theory extends readily if the algebra is a Frobenius algebra. Using matric methods introduced by Ward (Duke Math. J. vol. 7 (1940)), the author obtains the generalized Cauchy-Riemann equations as necessary and sufficient conditions for analyticity of a function in the algebra A of order n. The generalized Laplace equations are then obtained as necessary and sufficient conditions that a function u of n variables shall be a component of an analytic function. One form of the generalized Laplace equations is equivalent to the statement that the Hessian matrix of u shall be the parastrophic matrix of some number of A. (Received December 10, 1946.)

ANALYSIS

120. Richard Bellman: *On the boundedness of solutions of nonlinear differential and difference equations.*

The purpose of the author is to discuss the behavior of solutions of the nonlinear system of differential equations: $dx_i/dt=\sum_{j=1}^{N}a_{ij}x_j+f_i(x_1, x_2, \cdots, x_N, t)$, $i=1, 2, \cdots, n$, as $t \rightarrow \infty$, under various restrictions upon the matrix (a_{ij}), the functions $f_i(x, t)$, and the initial values. The more general case where the right-hand side contains the derivatives dx_i/dt is also considered. The three methods used are the method of successive approximations, the fixed-point method due to Birkhoff and Kellogg, and the method of approximating to a differential equation by a difference equation. Analogous results are derived for nonlinear difference equations. (Received January 22, 1947.)

121. Stefan Bergman: *Functions satisfying linear partial differential equations and their properties.*

The author investigates functions $\psi(\theta, \mathrm{H})$ of two real variables which satisfy the equation $(-\mathrm{H})^s\psi_{\theta\theta}+\psi_{\mathrm{HH}}=0$ for $\mathrm{H}\leqq 0$, and $(-\mathrm{H})^s\psi_{\theta\theta}+\psi_{\mathrm{HH}}=0$, for $\mathrm{H}\geqq 0$, $s>-2$, $\psi_{\theta\theta}\equiv(\partial^2\psi/\partial\theta^2)$, $\cdots$. In this paper the initial-value problem is considered. To this end $\psi(\theta, \mathrm{H})$ is expressed in terms of $T^{(0)}(\theta)$ and $T^{(1)}(\theta)$, the prescribed values of ψ and $\partial\psi/\partial\mathrm{H}$ respectively on the line $\mathrm{H}=0$. It is shown that the function ψ satisfying the conditions $\psi(\theta, 0)=0$, $\psi_{\mathrm{H}}(\theta, 0)\equiv[\partial(\theta, \mathrm{H})/\partial\mathrm{H}]_{\mathrm{H}=0}=T^{(1)}(\theta)$, $\theta^{(0)}\leqq\theta\leqq\theta^{(1)}$, where $T^{(1)}(\theta)$ is an analytic function of the real variable θ, can be written in the form $2\pi^2 i(2+s)\psi(\theta, \mathrm{H}) = \mathrm{H}\int_{\chi=0}^{1}\int_{t=0}^{\pi}\int_C(1-\chi)^{(s+2)^{-1}}T^{(1)}(\vartheta)dt\,d\chi d\vartheta/(\vartheta-\theta)[1+4(2+s)^{-2}(\vartheta-\theta)^{-2}(-\mathrm{H})^{s+2}\chi\sin^2 t]$, $\mathrm{H}<0$, and a similar expression holds for $\mathrm{H}>0$. C is a simple closed curve in the regularity domain of $T^{(1)}(\vartheta)$ (considered as a function of the complex variable $\vartheta=\theta+i\Theta$), which curve includes the interval $\theta^{(0)}\leqq\theta\leqq\theta^{(1)}$ of the real axis. An analogous formula holds for the solution ψ satisfying the condition $\psi(\theta, 0)=T^{(0)}(\theta)$, $\psi_{\mathrm{H}}(\theta, 0)=0$. Using these formulae, the author investigates the connections which exist between the location and the nature of the singularities of $T^{(0)}(\vartheta)$ and