BOOK REVIEWS

Les systèmes différentiels extérieurs et leurs applications géométriques. (Actualités Scientifiques et Industrielles, no. 994.) By Elie Cartan. Paris, Hermann, 1945. 214 pp. 450 fr.

This book gives a revised account of lectures delivered in 1936–1937 at the Sorbonne. The content is based almost exclusively on the author's own outstanding contributions to the subject at the beginning of the century. In developing many of these old results, however, the author's present viewpoint is new as well as stimulating. The whole treatment, even at the few points where it touches upon contemporary work by other writers, bears the stamp of the author's individuality.

The fundamental calculus employed is Grassmann algebra. Cartan's manner of regarding this discipline might be described as follows. The indeterminates are differentials and the polynomials are forms. Interest centers in skew-symmetric (necessarily multilinear) forms, which can be written

$$F = a_{i_1\cdots i_p} \overset{i_1}{u_1} \cdots \overset{i_p}{u_p}, \tag{1}$$

where the coefficients a are skew-symmetric in every pair of indices and where the summation convention is being used. An equivalent expression for F is

$$F = \frac{1}{p!} a_{i_1\cdots i_p} \begin{vmatrix} \overset{i_1}{u_1} & \cdots & \overset{i_p}{u_1} \\ \cdot & \cdot & \cdot \\ \overset{i_1}{u_p} & \cdots & \overset{i_p}{u_p} \end{vmatrix},$$

where the a's are the same as in (1). The exterior product of F by the form $G = b_{i_{p+1}\cdots i_{p+q}} \overset{i_{p+1}}{u_{p+1}} \cdots \overset{i_{p+q}}{u_{p+q}}$ is the skew-symmetric form defined by

$$[FG] = c_{i_1\cdots i_{p+q}} \overset{i_1}{u_1} \cdots \overset{i_{p+q}}{u_{p+q}},$$

where the c's are found by applying all $(p+q)!$ signed permutations to the subscripts on $a_{i_1\cdots i_p} b_{i_{p+1}\cdots i_{p+q}}$ and adding. The definition of product applied to $\overset{i_1}{u_1}, \cdots, \overset{i_p}{u_p}$ gives

$$[\overset{i_1}{u_1} \cdots \overset{i_p}{u_p}] = \begin{vmatrix} \overset{i_1}{u_1} & \cdots & \overset{i_p}{u_1} \\ \cdot & \cdot & \cdot \\ \overset{i_1}{u_p} & \cdots & \overset{i_p}{u_p} \end{vmatrix},$$